CHAPITRE 4 :

```
> restart;
> P:=-y+x*y;
```

$$P := -y + x\,y$$

```
> Q:=x+y^2;
```

$$Q := x + y^2$$

```
> Q*(a[2,0]*x^2+a[1,1]*x*y+a[0,2]*y^2)-P*(b[2,0]*x^2+b[1,1]*x*y+b[
  0,2]*y^2);
```

$$(x+y^2)\,(a_{2,0}\,x^2 + a_{1,1}\,x\,y + a_{0,2}\,y^2) - (-y + x\,y)\,(b_{2,0}\,x^2 + b_{1,1}\,x\,y + b_{0,2}\,y^2)$$

```
> H:=simplify(%);
```

$$H := a_{2,0}\,x^3 + a_{1,1}\,x^2\,y + x\,a_{0,2}\,y^2 + y^2\,a_{2,0}\,x^2 + y^3\,a_{1,1}\,x + y^4\,a_{0,2} + y\,b_{2,0}\,x^2 + b_{1,1}\,x\,y^2 + b_{0,2}\,y^3 - x^3\,y\,b_{2,0} - x^2\,y^2\,b_{1,1} - x\,y^3\,b_{0,2}$$

```
> x*(b[2,0]*x^2+b[1,1]*x*y+b[0,2]*y^2)-y*(a[2,0]*x^2+a[1,1]*x*y+a[
  0,2]*y^2);
```

$$x\,(b_{2,0}\,x^2 + b_{1,1}\,x\,y + b_{0,2}\,y^2) - y\,(a_{2,0}\,x^2 + a_{1,1}\,x\,y + a_{0,2}\,y^2)$$

```
> simplify(%);
```

$$b_{2,0}\,x^3 + b_{1,1}\,x^2\,y + x\,b_{0,2}\,y^2 - y\,a_{2,0}\,x^2 - a_{1,1}\,x\,y^2 - a_{0,2}\,y^3$$

```
> simplify(subs(x=rho*cos(phi),y=rho*sin(phi),H/(2*R*(x^2+y^2)^(3/
  2))));
```

$$\frac{1}{2}\,\mathrm{csgn}(\rho)\,(-\sin(\phi)\cos(\phi)^2\,b_{0,2} - \rho\cos(\phi)^3\sin(\phi)\,b_{2,0} + a_{1,1}\cos(\phi)^2\sin(\phi) + \sin(\phi)\,b_{2,0}\cos(\phi)^2 + \sin(\phi)\cos(\phi)^3\,\rho\,b_{0,2} + \sin(\phi)\,\rho\,a_{1,1}\cos(\phi) - \sin(\phi)\,\rho\cos(\phi)\,b_{0,2} + \sin(\phi)\,b_{0,2} - \sin(\phi)\cos(\phi)^3\,\rho\,a_{1,1} + b_{1,1}\cos(\phi) + \rho\,a_{0,2} - 2\,\rho\,a_{0,2}\cos(\phi)^2 + a_{2,0}\cos(\phi)^3 + \cos(\phi)^4\,\rho\,b_{1,1} - \cos(\phi)^3\,a_{0,2} - \cos(\phi)^3\,b_{1,1} + \cos(\phi)^4\,\rho\,a_{0,2} + \cos(\phi)\,a_{0,2} + \rho\,a_{2,0}\cos(\phi)^2 - \rho\cos(\phi)^2\,b_{1,1} - \cos(\phi)^4\,\rho\,a_{2,0})\,/\,R$$

```
> subs(rho=1/(R^2+cos(phi)),%);
```

$$\frac{1}{2}\,\mathrm{csgn}\left(\frac{1}{R^2+\cos(\phi)}\right)\left(-\sin(\phi)\cos(\phi)^2\,b_{0,2} - \frac{\cos(\phi)^3\sin(\phi)\,b_{2,0}}{R^2+\cos(\phi)} + a_{1,1}\cos(\phi)^2\sin(\phi) + \sin(\phi)\,b_{2,0}\cos(\phi)^2 + \frac{\sin(\phi)\cos(\phi)^3\,b_{0,2}}{R^2+\cos(\phi)} + \frac{\sin(\phi)\,a_{1,1}\cos(\phi)}{R^2+\cos(\phi)} - \frac{\sin(\phi)\cos(\phi)\,b_{0,2}}{R^2+\cos(\phi)} + \sin(\phi)\,b_{0,2} - \frac{\sin(\phi)\cos(\phi)^3\,a_{1,1}}{R^2+\cos(\phi)} + b_{1,1}\cos(\phi) + \frac{a_{0,2}}{R^2+\cos(\phi)} - \frac{2\,a_{0,2}\cos(\phi)^2}{R^2+\cos(\phi)} + a_{2,0}\cos(\phi)^3 + \frac{\cos(\phi)^4\,b_{1,1}}{R^2+\cos(\phi)} - \cos(\phi)^3\,a_{0,2} - \cos(\phi)^3\,b_{1,1} + \frac{\cos(\phi)^4\,a_{0,2}}{R^2+\cos(\phi)} + \cos(\phi)\,a_{0,2}\right)$$

$$+\frac{a_{2,0}\cos(\phi)^2}{R^2+\cos(\phi)}-\frac{\cos(\phi)^2 b_{1,1}}{R^2+\cos(\phi)}-\frac{\cos(\phi)^4 a_{2,0}}{R^2+\cos(\phi)}\Bigg)/R$$

```
> simplify((1/(2*Pi))*%);
```

$$-\frac{1}{4}\operatorname{csgn}\left(\frac{1}{R^2+\cos(\phi)}\right)(a_{0,2}\cos(\phi)^2-a_{0,2}-\sin(\phi)b_{0,2}R^2-\cos(\phi)a_{0,2}R^2+\cos(\phi)^3 b_{1,1}R^2$$
$$+\cos(\phi)^3 a_{0,2}R^2-a_{2,0}\cos(\phi)^3 R^2-a_{2,0}\cos(\phi)^2-b_{1,1}\cos(\phi)R^2+\sin(\phi)\cos(\phi)^2 b_{0,2}R^2$$
$$-a_{1,1}\cos(\phi)^2\sin(\phi)R^2-\sin(\phi)b_{2,0}\cos(\phi)^2 R^2-\sin(\phi)a_{1,1}\cos(\phi))\Big/(\pi R(R^2+\cos(\phi)))$$

```
> int(1/(4*Pi)*(b[1,1]*cos(phi)*R^2+a[0,2]+a[2,0]*cos(phi)^3*R^2+c
  os(phi)*a[0,2]*R^2-cos(phi)^2*a[0,2]+a[2,0]*cos(phi)^2-cos(phi)^
  3*b[1,1]*R^2-cos(phi)^3*a[0,2]*R^2)/(R^2+cos(phi))/R,phi=0..2*Pi
  );
```

$$-\frac{1}{4}\left(4R^4\operatorname{csgn}\left(\frac{R^2-1}{\sqrt{R^4-1}}\right)a_{0,2}-2R^8\operatorname{csgn}\left(\frac{R^2-1}{\sqrt{R^4-1}}\right)b_{1,1}+2R^4\operatorname{csgn}\left(\frac{R^2-1}{\sqrt{R^4-1}}\right)b_{1,1}\right.$$
$$+2R^8\operatorname{csgn}\left(\frac{R^2-1}{\sqrt{R^4-1}}\right)a_{2,0}-2R^8\operatorname{csgn}\left(\frac{R^2-1}{\sqrt{R^4-1}}\right)a_{0,2}-3R^2 a_{0,2}\sqrt{R^4-1}$$
$$+2R^6 a_{0,2}\sqrt{R^4-1}-2R^4\operatorname{csgn}\left(\frac{R^2-1}{\sqrt{R^4-1}}\right)a_{2,0}-R^2 b_{1,1}\sqrt{R^4-1}+2R^6 b_{1,1}\sqrt{R^4-1}$$
$$\left.+R^2 a_{2,0}\sqrt{R^4-1}-2R^6 a_{2,0}\sqrt{R^4-1}-2\operatorname{csgn}\left(\frac{R^2-1}{\sqrt{R^4-1}}\right)a_{0,2}\right)\Big/(R\sqrt{R^4-1})$$

```
> simplify(%);
```

$$-\frac{1}{4}\left(4R^4\operatorname{csgn}\left(\frac{R^2-1}{\sqrt{R^4-1}}\right)a_{0,2}-2R^8\operatorname{csgn}\left(\frac{R^2-1}{\sqrt{R^4-1}}\right)b_{1,1}+2R^4\operatorname{csgn}\left(\frac{R^2-1}{\sqrt{R^4-1}}\right)b_{1,1}\right.$$
$$+2R^8\operatorname{csgn}\left(\frac{R^2-1}{\sqrt{R^4-1}}\right)a_{2,0}-2R^8\operatorname{csgn}\left(\frac{R^2-1}{\sqrt{R^4-1}}\right)a_{0,2}-3R^2 a_{0,2}\sqrt{R^4-1}$$
$$+2R^6 a_{0,2}\sqrt{R^4-1}-2R^4\operatorname{csgn}\left(\frac{R^2-1}{\sqrt{R^4-1}}\right)a_{2,0}-R^2 b_{1,1}\sqrt{R^4-1}+2R^6 b_{1,1}\sqrt{R^4-1}$$
$$\left.+R^2 a_{2,0}\sqrt{R^4-1}-2R^6 a_{2,0}\sqrt{R^4-1}-2\operatorname{csgn}\left(\frac{R^2-1}{\sqrt{R^4-1}}\right)a_{0,2}\right)\Big/(R\sqrt{R^4-1})$$

```
> 1/4*(2*R^6*a[2,0]*(R^4-1)^(1/2)+3*R^2*a[0,2]*(R^4-1)^(1/2)+2*R^8
  *b[1,1]-2*R^6*b[1,1]*(R^4-1)^(1/2)+2*R^8*a[0,2]-R^2*a[2,0]*(R^4-
  1)^(1/2)-2*R^4*b[1,1]+R^2*b[1,1]*(R^4-1)^(1/2)+2*R^4*a[2,0]-2*R^
  6*a[0,2]*(R^4-1)^(1/2)+2*a[0,2]-2*R^8*a[2,0]-4*R^4*a[0,2])/(R^4-
  1)^(1/2)/R;
```

$$\frac{1}{4}(2R^6 a_{2,0}\sqrt{R^4-1}+3R^2 a_{0,2}\sqrt{R^4-1}+2R^8 b_{1,1}-2R^6 b_{1,1}\sqrt{R^4-1}+2R^8 a_{0,2}$$
$$-R^2 a_{2,0}\sqrt{R^4-1}-2R^4 b_{1,1}+R^2 b_{1,1}\sqrt{R^4-1}+2R^4 a_{2,0}-2R^6 a_{0,2}\sqrt{R^4-1}+2a_{0,2}$$

$$-2R^8 a_{2,0} - 4R^4 a_{0,2}) \Big/ (\sqrt{R^4-1}\, R)$$

```
> expand(%);
```

$$\frac{1}{2}R^5 a_{2,0} + \frac{3}{4}R\,a_{0,2} + \frac{1}{2}\frac{R^7 b_{1,1}}{\sqrt{R^4-1}} - \frac{1}{2}R^5 b_{1,1} + \frac{1}{2}\frac{R^7 a_{0,2}}{\sqrt{R^4-1}} - \frac{1}{4}R\,a_{2,0} - \frac{1}{2}\frac{R^3 b_{1,1}}{\sqrt{R^4-1}} + \frac{1}{4}R\,b_{1,1}$$
$$+\frac{1}{2}\frac{R^3 a_{2,0}}{\sqrt{R^4-1}} - \frac{1}{2}R^5 a_{0,2} + \frac{1}{2}\frac{a_{0,2}}{\sqrt{R^4-1}\,R} - \frac{1}{2}\frac{R^7 a_{2,0}}{\sqrt{R^4-1}} - \frac{R^3 a_{0,2}}{\sqrt{R^4-1}}$$

```
> F1:=normal(%);
```

$$F1 := -\frac{1}{4}(-2R^6 a_{2,0}\sqrt{R^4-1} - 3R^2 a_{0,2}\sqrt{R^4-1} - 2R^8 b_{1,1} + 2R^6 b_{1,1}\sqrt{R^4-1} - 2R^8 a_{0,2}$$
$$+R^2 a_{2,0}\sqrt{R^4-1} + 2R^4 b_{1,1} - R^2 b_{1,1}\sqrt{R^4-1} - 2R^4 a_{2,0} + 2R^6 a_{0,2}\sqrt{R^4-1} - 2a_{0,2}$$
$$+2R^8 a_{2,0} + 4R^4 a_{0,2}) \Big/ (R\sqrt{R^4-1})$$

```
> collect(expand(%),R);
```

$$\left(\frac{1}{2}\frac{b_{1,1}}{\sqrt{R^4-1}} - \frac{1}{2}\frac{a_{2,0}}{\sqrt{R^4-1}} + \frac{1}{2}\frac{a_{0,2}}{\sqrt{R^4-1}}\right)R^7 + \left(\frac{1}{2}a_{2,0} - \frac{1}{2}b_{1,1} - \frac{1}{2}a_{0,2}\right)R^5$$
$$+\left(\frac{1}{2}\frac{a_{2,0}}{\sqrt{R^4-1}} - \frac{a_{0,2}}{\sqrt{R^4-1}} - \frac{1}{2}\frac{b_{1,1}}{\sqrt{R^4-1}}\right)R^3 + \left(\frac{3}{4}a_{0,2} + \frac{1}{4}b_{1,1} - \frac{1}{4}a_{2,0}\right)R + \frac{1}{2}\frac{a_{0,2}}{\sqrt{R^4-1}\,R}$$

```
> normal(-1/2*1/(R^4-1)^(1/2)*R^7*a[2,0]+1/2*1/(R^4-1)^(1/2)*R^7*b
[1,1]+1/2*1/(R^4-1)^(1/2)*R^7*a[0,2]+1/2*1/(R^4-1)^(1/2)*R^3*a[2
,0]-1/(R^4-1)^(1/2)*R^3*a[0,2]-1/2*1/(R^4-1)^(1/2)*R^3*b[1,1]+1/
2*1/(R^4-1)^(1/2)*a[0,2]/R);
```

$$\frac{1}{2}\frac{-R^8 a_{2,0} + R^8 b_{1,1} + R^8 a_{0,2} + R^4 a_{2,0} - 2R^4 a_{0,2} - R^4 b_{1,1} + a_{0,2}}{R\sqrt{R^4-1}}$$

```
> collect(1/2*R^5*a[2,0]-1/2*R^5*b[1,1]-1/2*R^5*a[0,2]+3/4*R*a[0,2
]+1/4*R*b[1,1]-1/4*R*a[2,0],R);
```

$$\left(\frac{1}{2}a_{2,0} - \frac{1}{2}b_{1,1} - \frac{1}{2}a_{0,2}\right)R^5 + \left(\frac{3}{4}a_{0,2} + \frac{1}{4}b_{1,1} - \frac{1}{4}a_{2,0}\right)R$$

```
> Int((cos(phi)^2*sin(phi)^2)/(R^2+cos(phi)),phi=0..2*Pi)=int((cos
(phi)^2*sin(phi)^2)/(R^2+cos(phi)),phi=0..2*Pi);
```

$$\int_0^{2\pi} \frac{\cos(\phi)^2 \sin(\phi)^2}{R^2 + \cos(\phi)}\, d\phi =$$
$$-\frac{R^2\pi\left(-2R^2\,\mathrm{csgn}\left(\frac{R^2-1}{\sqrt{R^4-1}}\right) + 2R^6\,\mathrm{csgn}\left(\frac{R^2-1}{\sqrt{R^4-1}}\right) - 2R^4\sqrt{R^4-1} + \sqrt{R^4-1}\right)}{\sqrt{R^4-1}}$$

```
> expand(R^2*Pi*(-(R^4-1)^(1/2)+2*R^4*(R^4-1)^(1/2)-2*R^6+2*R^2)/(
R^4-1)^(1/2));
```

$$-R^2\,\pi + 2\,R^6\,\pi - \frac{2\,R^8\,\pi}{\sqrt{R^4-1}} + \frac{2\,R^4\,\pi}{\sqrt{R^4-1}}$$

```
> Int((sin(phi)^4)/(R^2+cos(phi)),phi=0..2*Pi)=int((sin(phi)^4)/(R
  ^2+cos(phi)),phi=0..2*Pi);
```

$$\int_0^{2\,\pi} \frac{\sin(\phi)^4}{R^2+\cos(\phi)}\,d\phi = -\pi\left(-2\,R^8\,\mathrm{csgn}\left(\frac{R^2-1}{\sqrt{R^4-1}}\right) + 2\,R^6\sqrt{R^4-1} - 3\,R^2\sqrt{R^4-1}\right.$$
$$\left. + 4\,R^4\,\mathrm{csgn}\left(\frac{R^2-1}{\sqrt{R^4-1}}\right) - 2\,\mathrm{csgn}\left(\frac{R^2-1}{\sqrt{R^4-1}}\right)\right) \Big/ \sqrt{R^4-1}$$

```
> expand(Pi*(3*R^2*(R^4-1)^(1/2)-2*R^6*(R^4-1)^(1/2)+2*R^8-4*R^4+2
  )/(R^4-1)^(1/2));
```

$$3\,R^2\,\pi - 2\,R^6\,\pi + \frac{2\,R^8\,\pi}{\sqrt{R^4-1}} - \frac{4\,R^4\,\pi}{\sqrt{R^4-1}} + \frac{2\,\pi}{\sqrt{R^4-1}}$$

```
> restart;
```

```
> F1 :=
  -1/4*(-2*R^6*a[2,0]*(R^4-1)^(1/2)-3*R^2*a[0,2]*(R^4-1)^(1/2)-2*R
  ^8*b[1,1]+2*R^6*b[1,1]*(R^4-1)^(1/2)-2*R^8*a[0,2]+R^2*a[2,0]*(R^
  4-1)^(1/2)+2*R^4*b[1,1]-R^2*b[1,1]*(R^4-1)^(1/2)-2*R^4*a[2,0]+2*
  R^6*a[0,2]*(R^4-1)^(1/2)-2*a[0,2]+2*R^8*a[2,0]+4*R^4*a[0,2])/(R^
  4-1)^(1/2)/R;
```

$$F1 := -\frac{1}{4}\left(-2\,R^6\,a_{2,0}\sqrt{R^4-1} - 3\,R^2\,a_{0,2}\sqrt{R^4-1} - 2\,R^8\,b_{1,1} + 2\,R^6\,b_{1,1}\sqrt{R^4-1} - 2\,R^8\,a_{0,2}\right.$$
$$+ R^2\,a_{2,0}\sqrt{R^4-1} + 2\,R^4\,b_{1,1} - R^2\,b_{1,1}\sqrt{R^4-1} - 2\,R^4\,a_{2,0} + 2\,R^6\,a_{0,2}\sqrt{R^4-1} - 2\,a_{0,2}$$
$$\left. + 2\,R^8\,a_{2,0} + 4\,R^4\,a_{0,2}\right) \Big/ \left(\sqrt{R^4-1}\,R\right)$$

```
> expand(%);
```

$$\frac{1}{2}R^5\,a_{2,0} + \frac{3}{4}R\,a_{0,2} + \frac{1}{2}\frac{R^7\,b_{1,1}}{\sqrt{R^4-1}} - \frac{1}{2}R^5\,b_{1,1} + \frac{1}{2}\frac{R^7\,a_{0,2}}{\sqrt{R^4-1}} - \frac{1}{4}R\,a_{2,0} - \frac{1}{2}\frac{R^3\,b_{1,1}}{\sqrt{R^4-1}} + \frac{1}{4}R\,b_{1,1}$$
$$+ \frac{1}{2}\frac{R^3\,a_{2,0}}{\sqrt{R^4-1}} - \frac{1}{2}R^5\,a_{0,2} + \frac{1}{2}\frac{a_{0,2}}{\sqrt{R^4-1}\,R} - \frac{1}{2}\frac{R^7\,a_{2,0}}{\sqrt{R^4-1}} - \frac{R^3\,a_{0,2}}{\sqrt{R^4-1}}$$

```
> collect(%,R);
```

$$\left(\frac{1}{2}\frac{b_{1,1}}{\sqrt{R^4-1}} - \frac{1}{2}\frac{a_{2,0}}{\sqrt{R^4-1}} + \frac{1}{2}\frac{a_{0,2}}{\sqrt{R^4-1}}\right)R^7 + \left(-\frac{1}{2}b_{1,1} + \frac{1}{2}a_{2,0} - \frac{1}{2}a_{0,2}\right)R^5$$
$$+ \left(\frac{1}{2}\frac{a_{2,0}}{\sqrt{R^4-1}} - \frac{a_{0,2}}{\sqrt{R^4-1}} - \frac{1}{2}\frac{b_{1,1}}{\sqrt{R^4-1}}\right)R^3 + \left(\frac{3}{4}a_{0,2} + \frac{1}{4}b_{1,1} - \frac{1}{4}a_{2,0}\right)R + \frac{1}{2}\frac{a_{0,2}}{\sqrt{R^4-1}\,R}$$

```
> (2*a[2,0]-2*b[1,1]-2*a[0,2])*(1+w^2)^3*w+(-a[2,0]+b[1,1]+3*a[0,2
  ])*(1+w^2)*(1-w^2)^2*w+(-a[2,0]+b[1,1]+a[0,2])*(1+w^2)^4+(a[2,0]
  -b[1,1]-2*a[0,2])*(1+w^2)^2*(1-w^2)^2+a[0,2]*(1-w^2)^4;
```

$$(2\,a_{2,0}-2\,b_{1,1}-2\,a_{0,2})\,(1+w^2)^3\,w+(-a_{2,0}+b_{1,1}+3\,a_{0,2})\,(1+w^2)\,(1-w^2)^2\,w$$
$$+(-a_{2,0}+b_{1,1}+a_{0,2})\,(1+w^2)^4+(a_{2,0}-b_{1,1}-2\,a_{0,2})\,(1+w^2)^2\,(1-w^2)^2+a_{0,2}\,(1-w^2)^4$$

```
> simplify(expand(%));
```

$$a_{0,2}\,w^7-9\,a_{0,2}\,w^5-9\,a_{0,2}\,w^3+w\,a_{0,2}-b_{1,1}\,w^7-7\,b_{1,1}\,w^5-7\,b_{1,1}\,w^3-w\,b_{1,1}+a_{2,0}\,w^7$$
$$+7\,a_{2,0}\,w^5+7\,a_{2,0}\,w^3+w\,a_{2,0}-4\,a_{2,0}\,w^2-8\,a_{2,0}\,w^4-4\,a_{2,0}\,w^6+4\,b_{1,1}\,w^2+8\,b_{1,1}\,w^4$$
$$+4\,b_{1,1}\,w^6+16\,a_{0,2}\,w^4$$

```
> collect(%,w);
```

$$(a_{0,2}-b_{1,1}+a_{2,0})\,w^7+(-4\,a_{2,0}+4\,b_{1,1})\,w^6+(-7\,b_{1,1}+7\,a_{2,0}-9\,a_{0,2})\,w^5$$
$$+(16\,a_{0,2}-8\,a_{2,0}+8\,b_{1,1})\,w^4+(-7\,b_{1,1}+7\,a_{2,0}-9\,a_{0,2})\,w^3+(-4\,a_{2,0}+4\,b_{1,1})\,w^2$$
$$+(a_{0,2}-b_{1,1}+a_{2,0})\,w$$

```
> factor(%);
```

$$w\,(w-1)^4\,(a_{0,2}\,w^2-b_{1,1}\,w^2+a_{2,0}\,w^2+4\,w\,a_{0,2}+a_{0,2}-b_{1,1}+a_{2,0})$$

```
> solve(%,w);
```

$$0,\ 1,\ 1,\ 1,\ 1,\ -\frac{2\,a_{0,2}+\sqrt{3\,a_{0,2}^{\,2}+2\,a_{0,2}\,b_{1,1}-2\,a_{0,2}\,a_{2,0}-b_{1,1}^{\,2}+2\,b_{1,1}\,a_{2,0}-a_{2,0}^{\,2}}}{a_{0,2}-b_{1,1}+a_{2,0}},$$
$$\frac{-2\,a_{0,2}+\sqrt{3\,a_{0,2}^{\,2}+2\,a_{0,2}\,b_{1,1}-2\,a_{0,2}\,a_{2,0}-b_{1,1}^{\,2}+2\,b_{1,1}\,a_{2,0}-a_{2,0}^{\,2}}}{a_{0,2}-b_{1,1}+a_{2,0}}$$

Exemple 1 :

```
> restart;
> G1:=1/(4*Pi*R)*((13/8)*(-R^2*Pi+2*R^6*Pi-2*R^8*Pi/(R^4-1)^(1/2)+
  2*R^4*Pi/(R^4-1)^(1/2))-(5/8)*(3*R^2*Pi-2*R^6*Pi+2*R^8*Pi/(R^4-1
  )^(1/2)-4*R^4*Pi/(R^4-1)^(1/2)+2*Pi/(R^4-1)^(1/2)));
```

$$G1:=\frac{-\frac{7\,R^2\,\pi}{2}+\frac{9\,R^6\,\pi}{2}-\frac{9\,R^8\,\pi}{2\sqrt{R^4-1}}+\frac{23\,R^4\,\pi}{4\sqrt{R^4-1}}-\frac{5\,\pi}{4\sqrt{R^4-1}}}{4\,\pi\,R}$$

```
> solve(%,R);
```

$$\frac{\sqrt{15}}{3},\ -\frac{\sqrt{15}}{3}$$

```
> simplify(diff(G1,R));
```

$$\frac{-90\,R^8+90\,R^6\sqrt{R^4-1}+59\,R^4-14\,R^2\sqrt{R^4-1}+5}{16\,R^2\sqrt{R^4-1}}$$

```
> simplify(subs(R=sqrt(15)/3,%));
```

$$\frac{-1}{32}$$

```
> solve((1+w^2)/(1-w^2)=15/9,w);
```

$$\frac{-1}{2}, \frac{1}{2}$$

```
> restart;
```

```
> -1/2*(sin(phi)*cos(phi)^2*b[0,2]+1/(R^2+cos(phi))*cos(phi)^3*sin
  (phi)*b[2,0]-a[1,1]*cos(phi)^2*sin(phi)-sin(phi)*b[2,0]*cos(phi)
  ^2-sin(phi)*cos(phi)^3/(R^2+cos(phi))*b[0,2]-sin(phi)/(R^2+cos(p
  hi))*a[1,1]*cos(phi)+sin(phi)/(R^2+cos(phi))*cos(phi)*b[0,2]-sin
  (phi)*b[0,2]+sin(phi)*cos(phi)^3/(R^2+cos(phi))*a[1,1]-b[1,1]*co
  s(phi)-1/(R^2+cos(phi))*a[0,2]+2/(R^2+cos(phi))*a[0,2]*cos(phi)^
  2-a[2,0]*cos(phi)^3-cos(phi)^4/(R^2+cos(phi))*b[1,1]+cos(phi)^3*
  a[0,2]+cos(phi)^3*b[1,1]-cos(phi)^4/(R^2+cos(phi))*a[0,2]-cos(ph
  i)*a[0,2]-1/(R^2+cos(phi))*a[2,0]*cos(phi)^2+1/(R^2+cos(phi))*co
  s(phi)^2*b[1,1]+cos(phi)^4/(R^2+cos(phi))*a[2,0])/R;
```

$$-\frac{1}{2}\left(\sin(\phi)\cos(\phi)^2 b_{0,2}+\frac{\cos(\phi)^3\sin(\phi)\,b_{2,0}}{R^2+\cos(\phi)}-a_{1,1}\cos(\phi)^2\sin(\phi)-\sin(\phi)\,b_{2,0}\cos(\phi)^2\right.$$
$$-\frac{\sin(\phi)\cos(\phi)^3 b_{0,2}}{R^2+\cos(\phi)}-\frac{\sin(\phi)\,a_{1,1}\cos(\phi)}{R^2+\cos(\phi)}+\frac{\sin(\phi)\cos(\phi)\,b_{0,2}}{R^2+\cos(\phi)}-\sin(\phi)\,b_{0,2}$$
$$+\frac{\sin(\phi)\cos(\phi)^3 a_{1,1}}{R^2+\cos(\phi)}-b_{1,1}\cos(\phi)-\frac{a_{0,2}}{R^2+\cos(\phi)}+\frac{2\,a_{0,2}\cos(\phi)^2}{R^2+\cos(\phi)}-a_{2,0}\cos(\phi)^3$$
$$-\frac{\cos(\phi)^4 b_{1,1}}{R^2+\cos(\phi)}+\cos(\phi)^3 a_{0,2}+\cos(\phi)^3 b_{1,1}-\frac{\cos(\phi)^4 a_{0,2}}{R^2+\cos(\phi)}-\cos(\phi)\,a_{0,2}-\frac{a_{2,0}\cos(\phi)^2}{R^2+\cos(\phi)}$$
$$\left.+\frac{\cos(\phi)^2 b_{1,1}}{R^2+\cos(\phi)}+\frac{\cos(\phi)^4 a_{2,0}}{R^2+\cos(\phi)}\right)/R$$

```
> F11:=subs(a[2,0]=b[1,1],a[0,2]=0,phi=theta,%);
```

$$F11 := -\frac{1}{2}\left(\sin(\theta)\cos(\theta)^2 b_{0,2}+\frac{\cos(\theta)^3\sin(\theta)\,b_{2,0}}{R^2+\cos(\theta)}-a_{1,1}\cos(\theta)^2\sin(\theta)\right.$$
$$-\sin(\theta)\,b_{2,0}\cos(\theta)^2-\frac{\sin(\theta)\cos(\theta)^3 b_{0,2}}{R^2+\cos(\theta)}-\frac{\sin(\theta)\,a_{1,1}\cos(\theta)}{R^2+\cos(\theta)}+\frac{\sin(\theta)\cos(\theta)\,b_{0,2}}{R^2+\cos(\theta)}$$
$$\left.-\sin(\theta)\,b_{0,2}+\frac{\sin(\theta)\cos(\theta)^3 a_{1,1}}{R^2+\cos(\theta)}-b_{1,1}\cos(\theta)\right)/R$$

```
> int(F11,theta=0..phi);
```

$$\frac{1}{4}(R^2 a_{1,1}-R^2 b_{0,2}+R^2 b_{2,0}+2R^2 a_{1,1}\ln(R^2+\cos(\phi))-2R^2 b_{0,2}\ln(R^2+\cos(\phi))$$
$$-2\,a_{1,1}\cos(\phi)+2\,b_{1,1}\sin(\phi)+2R^4 a_{1,1}\cos(\phi)-R^2 a_{1,1}\cos(\phi)^2-R^2 b_{2,0}\cos(\phi)^2$$
$$-2R^6 b_{2,0}\ln(R^2+\cos(\phi))-2R^4 b_{0,2}\cos(\phi)+R^2 b_{0,2}\cos(\phi)^2+2R^6 b_{0,2}\ln(R^2+\cos(\phi))$$
$$+2R^4 b_{2,0}\cos(\phi)-2R^6 a_{1,1}\ln(R^2+\cos(\phi))+2R^2 b_{0,2}\ln(R^2+1)+2\,a_{1,1}$$
$$-2R^6 b_{0,2}\ln(R^2+1)-2R^2 a_{1,1}\ln(R^2+1)+2R^6 b_{2,0}\ln(R^2+1)+2R^6 a_{1,1}\ln(R^2+1)$$

$$-2\,R^4\,b_{2,0}-2\,R^4\,a_{1,1}+2\,R^4\,b_{0,2})/R$$

```
> y1:=expand(%);
```

$$y1:=\frac{1}{2}R^3\,a_{1,1}\cos(\phi)-\frac{1}{2}R\,b_{0,2}\ln(R^2+\cos(\phi))+\frac{1}{2}R\,a_{1,1}\ln(R^2+\cos(\phi))-\frac{1}{2}\frac{a_{1,1}\cos(\phi)}{R}$$
$$+\frac{1}{2}R^5\,b_{0,2}\ln(R^2+\cos(\phi))+\frac{1}{4}R\,b_{0,2}\cos(\phi)^2-\frac{1}{2}R^3\,b_{0,2}\cos(\phi)-\frac{1}{2}R^5\,b_{2,0}\ln(R^2+\cos(\phi))$$
$$-\frac{1}{4}R\,b_{2,0}\cos(\phi)^2+\frac{1}{2}R^3\,b_{2,0}\cos(\phi)-\frac{1}{2}R^5\,a_{1,1}\ln(R^2+\cos(\phi))-\frac{1}{4}R\,a_{1,1}\cos(\phi)^2$$
$$+\frac{1}{2}\frac{b_{1,1}\sin(\phi)}{R}+\frac{1}{2}R^3\,b_{0,2}+\frac{1}{4}R\,a_{1,1}-\frac{1}{4}R\,b_{0,2}+\frac{1}{2}\frac{a_{1,1}}{R}-\frac{1}{2}R^3\,b_{2,0}+\frac{1}{4}R\,b_{2,0}-\frac{1}{2}R^3\,a_{1,1}$$
$$+\frac{1}{2}R^5\,a_{1,1}\ln(R^2+1)+\frac{1}{2}R^5\,b_{2,0}\ln(R^2+1)+\frac{1}{2}R\,b_{0,2}\ln(R^2+1)-\frac{1}{2}R\,a_{1,1}\ln(R^2+1)$$
$$-\frac{1}{2}R^5\,b_{0,2}\ln(R^2+1)$$

```
> Int((cos(theta)^3*sin(theta))/(R^2+cos(theta)),theta=0..phi)=int
  ((cos(theta)^3*sin(theta))/(R^2+cos(theta)),theta=0..phi);
```

$$\int_0^{\phi}\frac{\cos(\theta)^3\sin(\theta)}{R^2+\cos(\theta)}\,d\theta=$$

$$R^4-\frac{R^2}{2}+\frac{1}{3}-R^6\ln(R^2+1)-\cos(\phi)\,R^4+\frac{1}{2}\cos(\phi)^2\,R^2-\frac{1}{3}\cos(\phi)^3+R^6\ln(R^2+\cos(\phi))$$

```
> Int((cos(theta)*sin(theta)^3)/(R^2+cos(theta)),theta=0..phi)=int
  ((cos(theta)*sin(theta)^3)/(R^2+cos(theta)),theta=0..phi);
```

$$\int_0^{\phi}\frac{\cos(\theta)\sin(\theta)^3}{R^2+\cos(\theta)}\,d\theta=-R^4+\frac{R^2}{2}+\frac{2}{3}-R^2\ln(R^2+1)+R^6\ln(R^2+1)+\cos(\phi)\,R^4$$
$$-\frac{1}{2}\cos(\phi)^2\,R^2+\frac{1}{3}\cos(\phi)^3-\cos(\phi)+R^2\ln(R^2+\cos(\phi))-R^6\ln(R^2+\cos(\phi))$$

```
> Int(1/(R^2+cos(phi)),phi=0..2*Pi)=int(1/(R^2+cos(phi)),phi=0..2*
  Pi);
```

$$\int_0^{2\pi}\frac{1}{R^2+\cos(\phi)}\,d\phi=\frac{2\,\mathrm{csgn}\left(\frac{R^2-1}{\sqrt{-1+R^4}}\right)\pi}{\sqrt{-1+R^4}}$$

```
> Int(cos(phi)/(R^2+cos(phi)),phi=0..2*Pi)=int(cos(phi)/(R^2+cos(p
  hi)),phi=0..2*Pi);
```

$$\int_0^{2\pi} \frac{\cos(\phi)}{R^2+\cos(\phi)}\,d\phi = -\frac{2\pi\left(R^2\,\mathrm{csgn}\left(\frac{R^2-1}{\sqrt{-1+R^4}}\right)-\sqrt{-1+R^4}\right)}{\sqrt{-1+R^4}}$$

```
> Int(cos(phi)^2/(R^2+cos(phi)),phi=0..2*Pi)=int(cos(phi)^2/(R^2+cos(phi)),phi=0..2*Pi);
```

$$\int_0^{2\pi} \frac{\cos(\phi)^2}{R^2+\cos(\phi)}\,d\phi = \frac{2R^2\pi\left(R^2\,\mathrm{csgn}\left(\frac{R^2-1}{\sqrt{-1+R^4}}\right)-\sqrt{-1+R^4}\right)}{\sqrt{-1+R^4}}$$

```
> Int(cos(phi)^3/(R^2+cos(phi)),phi=0..2*Pi)=int(cos(phi)^3/(R^2+cos(phi)),phi=0..2*Pi);
```

$$\int_0^{2\pi} \frac{\cos(\phi)^3}{R^2+\cos(\phi)}\,d\phi = -\frac{\pi\left(2R^6\,\mathrm{csgn}\left(\frac{R^2-1}{\sqrt{-1+R^4}}\right)-\sqrt{-1+R^4}-2R^4\sqrt{-1+R^4}\right)}{\sqrt{-1+R^4}}$$

```
> Int(cos(phi)^4/(R^2+cos(phi)),phi=0..2*Pi)=int(cos(phi)^4/(R^2+cos(phi)),phi=0..2*Pi);
```

$$\int_0^{2\pi} \frac{\cos(\phi)^4}{R^2+\cos(\phi)}\,d\phi = \frac{R^2\pi\left(2R^6\,\mathrm{csgn}\left(\frac{R^2-1}{\sqrt{-1+R^4}}\right)-\sqrt{-1+R^4}-2R^4\sqrt{-1+R^4}\right)}{\sqrt{-1+R^4}}$$

```
> Int(cos(phi)^5/(R^2+cos(phi)),phi=0..2*Pi)=int(cos(phi)^5/(R^2+cos(phi)),phi=0..2*Pi);
```

$$\int_0^{2\pi} \frac{\cos(\phi)^5}{R^2+\cos(\phi)}\,d\phi =$$

$$-\frac{1}{4}\frac{\pi\left(-3\sqrt{-1+R^4}-4R^4\sqrt{-1+R^4}+8R^{10}\,\mathrm{csgn}\left(\frac{R^2-1}{\sqrt{-1+R^4}}\right)-8R^8\sqrt{-1+R^4}\right)}{\sqrt{-1+R^4}}$$

```
> Int(cos(phi)^6/(R^2+cos(phi)),phi=0..2*Pi)=int(cos(phi)^6/(R^2+cos(phi)),phi=0..2*Pi);
```

$$\int_0^{2\pi} \frac{\cos(\phi)^6}{R^2+\cos(\phi)}\,d\phi =$$

$$\frac{1}{4}\frac{R^2\pi\left(-3\sqrt{-1+R^4}-4R^4\sqrt{-1+R^4}+8R^{10}\operatorname{csgn}\left(\frac{R^2-1}{\sqrt{-1+R^4}}\right)-8R^8\sqrt{-1+R^4}\right)}{\sqrt{-1+R^4}}$$

```
> Int(cos(phi)*sin(phi)^4/(R^2+cos(phi)),phi=0..2*Pi)=int(cos(phi)
*sin(phi)^4/(R^2+cos(phi)),phi=0..2*Pi);
```

$$\int_0^{2\pi}\frac{\cos(\phi)\sin(\phi)^4}{R^2+\cos(\phi)}\,d\phi=\frac{1}{4}\pi\left(3\sqrt{-1+R^4}-8R^{10}\operatorname{csgn}\left(\frac{R^2-1}{\sqrt{-1+R^4}}\right)+16R^6\operatorname{csgn}\left(\frac{R^2-1}{\sqrt{-1+R^4}}\right)-8R^2\operatorname{csgn}\left(\frac{R^2-1}{\sqrt{-1+R^4}}\right)+8R^8\sqrt{-1+R^4}-12R^4\sqrt{-1+R^4}\right)\Big/\sqrt{-1+R^4}$$

```
> expand(rhs(%));
```

$$\frac{3\pi}{4}-\frac{2\pi R^{10}\operatorname{csgn}\left(\frac{R^2}{\sqrt{-1+R^4}}-\frac{1}{\sqrt{-1+R^4}}\right)}{\sqrt{-1+R^4}}+\frac{4\pi R^6\operatorname{csgn}\left(\frac{R^2}{\sqrt{-1+R^4}}-\frac{1}{\sqrt{-1+R^4}}\right)}{\sqrt{-1+R^4}}-\frac{2\pi R^2\operatorname{csgn}\left(\frac{R^2}{\sqrt{-1+R^4}}-\frac{1}{\sqrt{-1+R^4}}\right)}{\sqrt{-1+R^4}}+2\pi R^8-3R^4\pi$$

```
> 3/4*Pi+2*Pi*R^8-2*Pi/(-1+R^4)^(1/2)*R^10-2*Pi/(-1+R^4)^(1/2)*R^2
+4*Pi/(-1+R^4)^(1/2)*R^6-3*R^4*Pi;
```

$$\frac{3\pi}{4}+2\pi R^8-\frac{2\pi R^{10}}{\sqrt{-1+R^4}}-\frac{2\pi R^2}{\sqrt{-1+R^4}}+\frac{4\pi R^6}{\sqrt{-1+R^4}}-3R^4\pi$$

```
> Int(cos(phi)^2*sin(phi)^4/(R^2+cos(phi)),phi=0..2*Pi)=int(cos(ph
i)^2*sin(phi)^4/(R^2+cos(phi)),phi=0..2*Pi);
```

$$\int_0^{2\pi}\frac{\cos(\phi)^2\sin(\phi)^4}{R^2+\cos(\phi)}\,d\phi=-\frac{1}{4}R^2\pi\left(3\sqrt{-1+R^4}-8R^{10}\operatorname{csgn}\left(\frac{R^2-1}{\sqrt{-1+R^4}}\right)+16R^6\operatorname{csgn}\left(\frac{R^2-1}{\sqrt{-1+R^4}}\right)-8R^2\operatorname{csgn}\left(\frac{R^2-1}{\sqrt{-1+R^4}}\right)+8R^8\sqrt{-1+R^4}-12R^4\sqrt{-1+R^4}\right)\Big/\sqrt{-1+R^4}$$

```
> expand(rhs(%));
```

$$-\frac{3\pi R^2}{4}+\frac{2\pi R^{12}\operatorname{csgn}\left(\frac{R^2}{\sqrt{-1+R^4}}-\frac{1}{\sqrt{-1+R^4}}\right)}{\sqrt{-1+R^4}}-\frac{4\pi R^8\operatorname{csgn}\left(\frac{R^2}{\sqrt{-1+R^4}}-\frac{1}{\sqrt{-1+R^4}}\right)}{\sqrt{-1+R^4}}$$

$$+\frac{2\,\pi\,R^4\,\mathrm{csgn}\left(\frac{R^2}{\sqrt{-1+R^4}}-\frac{1}{\sqrt{-1+R^4}}\right)}{\sqrt{-1+R^4}}-2\,\pi\,R^{10}+3\,\pi\,R^6$$

```
> -3/4*Pi*R^2-2*Pi*R^10+2*Pi/(-1+R^4)^(1/2)*R^12+2*Pi/(-1+R^4)^(1/
  2)*R^4-4*Pi/(-1+R^4)^(1/2)*R^8+3*Pi*R^6;
```

$$-\frac{3\,\pi\,R^2}{4}-2\,\pi\,R^{10}+\frac{2\,\pi\,R^{12}}{\sqrt{-1+R^4}}+\frac{2\,\pi\,R^4}{\sqrt{-1+R^4}}-\frac{4\,\pi\,R^8}{\sqrt{-1+R^4}}+3\,\pi\,R^6$$

```
> Int(cos(phi)^3*sin(phi)^2/(R^2+cos(phi)),phi=0..2*Pi)=int(cos(ph
  i)^3*sin(phi)^2/(R^2+cos(phi)),phi=0..2*Pi);
```

$$\int_0^{2\,\pi}\frac{\cos(\phi)^3\,\sin(\phi)^2}{R^2+\cos(\phi)}\,d\phi=\frac{1}{4}\,\pi\left(4\,R^4\sqrt{-1+R^4}+\sqrt{-1+R^4}+8\,R^{10}\,\mathrm{csgn}\left(\frac{R^2-1}{\sqrt{-1+R^4}}\right)\right.$$

$$\left.-8\,R^8\sqrt{-1+R^4}-8\,R^6\,\mathrm{csgn}\left(\frac{R^2-1}{\sqrt{-1+R^4}}\right)\right)\Big/\sqrt{-1+R^4}$$

```
> expand(rhs(%));
```

$$R^4\,\pi+\frac{\pi}{4}+\frac{2\,\pi\,R^{10}\,\mathrm{csgn}\left(\frac{R^2}{\sqrt{-1+R^4}}-\frac{1}{\sqrt{-1+R^4}}\right)}{\sqrt{-1+R^4}}-2\,\pi\,R^8$$

$$-\frac{2\,\pi\,R^6\,\mathrm{csgn}\left(\frac{R^2}{\sqrt{-1+R^4}}-\frac{1}{\sqrt{-1+R^4}}\right)}{\sqrt{-1+R^4}}$$

```
> 1/4*Pi-2*Pi/(-1+R^4)^(1/2)*R^6+2*Pi/(-1+R^4)^(1/2)*R^10+R^4*Pi-2
  *Pi*R^8;
```

$$\frac{\pi}{4}-\frac{2\,\pi\,R^6}{\sqrt{-1+R^4}}+\frac{2\,\pi\,R^{10}}{\sqrt{-1+R^4}}+R^4\,\pi-2\,\pi\,R^8$$

```
> Int((cos(phi)^4*sin(phi)^2)/(R^2+cos(phi)),phi=0..2*Pi)=int((cos
  (phi)^4*sin(phi)^2)/(R^2+cos(phi)),phi=0..2*Pi);
```

$$\int_0^{2\,\pi}\frac{\cos(\phi)^4\,\sin(\phi)^2}{R^2+\cos(\phi)}\,d\phi=-\frac{1}{4}\,R^2\,\pi$$

$$\left(-8\,R^8\sqrt{R^4-1}+\sqrt{R^4-1}+4\,R^4\sqrt{R^4-1}+8\,R^{10}\,\mathrm{csgn}\left(\frac{R^2-1}{\sqrt{R^4-1}}\right)-8\,R^6\,\mathrm{csgn}\left(\frac{R^2-1}{\sqrt{R^4-1}}\right)\right)$$

$$\Big/\sqrt{R^4-1}$$

```
> expand(rhs(%));
```

$$2\,R^{10}\,\pi-\frac{R^2\,\pi}{4}-R^6\,\pi-\frac{2\,R^{12}\,\pi\,\operatorname{csgn}\left(\frac{R^2}{\sqrt{R^4-1}}-\frac{1}{\sqrt{R^4-1}}\right)}{\sqrt{R^4-1}}+\frac{2\,R^8\,\pi\,\operatorname{csgn}\left(\frac{R^2}{\sqrt{R^4-1}}-\frac{1}{\sqrt{R^4-1}}\right)}{\sqrt{R^4-1}}$$

```
> 2*R^10*Pi-1/4*R^2*Pi-R^6*Pi-2*R^12*Pi/(R^4-1)^(1/2)+2*R^8*Pi/(R^
  4-1)^(1/2);
```

$$2\,R^{10}\,\pi-\frac{R^2\,\pi}{4}-R^6\,\pi-\frac{2\,R^{12}\,\pi}{\sqrt{R^4-1}}+\frac{2\,R^8\,\pi}{\sqrt{R^4-1}}$$

```
> restart;
> int((-3*sin(phi)^2*cos(phi)^2+cos(phi)^4)/(R^2+cos(phi)),phi=0..
  2*Pi);
```

$$\frac{2\,R^2\,\pi\left(-3\,R^2\,\operatorname{csgn}\left(\frac{R^2-1}{\sqrt{R^4-1}}\right)-4\,R^4\sqrt{R^4-1}+\sqrt{R^4-1}+4\,R^6\,\operatorname{csgn}\left(\frac{R^2-1}{\sqrt{R^4-1}}\right)\right)}{\sqrt{R^4-1}}$$

```
> expand(%);
```

$$-\frac{6\,R^4\,\operatorname{csgn}\left(\frac{R^2}{\sqrt{R^4-1}}-\frac{1}{\sqrt{R^4-1}}\right)\pi}{\sqrt{R^4-1}}-8\,R^6\,\pi+2\,R^2\,\pi+\frac{8\,R^8\,\operatorname{csgn}\left(\frac{R^2}{\sqrt{R^4-1}}-\frac{1}{\sqrt{R^4-1}}\right)\pi}{\sqrt{R^4-1}}$$

```
> with(student);
```

[D, *Diff*, *Doubleint*, *Int*, *Limit*, *Lineint*, *Product*, *Sum*, *Tripleint*, *changevar*, *completesquare*, *distance*, *equate*, *integrand*, *intercept*, *intparts*, *leftbox*, *leftsum*, *makeproc*, *middlebox*, *middlesum*, *midpoint*, *powsubs*, *rightbox*, *rightsum*, *showtangent*, *simpson*, *slope*, *summand*, *trapezoid*]

```
> -intparts(Int(cos(phi)^3*sin(phi)^2/(R^2+cos(phi))^2,phi=0..2*Pi
  ),cos(phi)^3*sin(phi));
```

$$\int_0^{2\,\pi}\frac{-3\,\cos(\phi)^2\,\sin(\phi)^2+\cos(\phi)^4}{R^2+\cos(\phi)}\,d\phi$$

```
> int((-3*sin(phi)^2*cos(phi)^2+cos(phi)^4)/(R^2+cos(phi)),phi=0..
  2*Pi);
```

$$\frac{2\,R^2\,\pi\left(-3\,R^2\,\operatorname{csgn}\left(\frac{R^2-1}{\sqrt{R^4-1}}\right)-4\,R^4\sqrt{R^4-1}+\sqrt{R^4-1}+4\,R^6\,\operatorname{csgn}\left(\frac{R^2-1}{\sqrt{R^4-1}}\right)\right)}{\sqrt{R^4-1}}$$

```
> 2*R^2*Pi*(-3*R^2+4*R^6+(R^4-1)^(1/2)-4*R^4*(R^4-1)^(1/2))/(R^4-1
  )^(1/2);
```

$$\frac{2\,R^2\,\pi\,(-3\,R^2+4\,R^6+\sqrt{R^4-1}-4\,R^4\sqrt{R^4-1})}{\sqrt{R^4-1}}$$

```
> -intparts(Int(cos(phi)*sin(phi)^4/(R^2+cos(phi))^2,phi=0..2*Pi),
  cos(phi)*sin(phi)^3);
```

$$\int_0^{2\pi} \frac{-\sin(\phi)^4+3\cos(\phi)^2\sin(\phi)^2}{R^2+\cos(\phi)}\,d\phi$$

```
> int((-sin(phi)^4+3*sin(phi)^2*cos(phi)^2)/(R^2+cos(phi)),phi=0..
  2*Pi);
```

$$2\,\pi\left(5\,R^4\,\mathrm{csgn}\left(\frac{R^2-1}{\sqrt{R^4-1}}\right)-\mathrm{csgn}\left(\frac{R^2-1}{\sqrt{R^4-1}}\right)+4\,R^6\sqrt{R^4-1}-3\,R^2\sqrt{R^4-1}\right.$$
$$\left.-4\,R^8\,\mathrm{csgn}\left(\frac{R^2-1}{\sqrt{R^4-1}}\right)\right)\Big/\sqrt{R^4-1}$$

```
> 2*Pi*(-1-4*R^8-3*R^2*(R^4-1)^(1/2)+5*R^4+4*R^6*(R^4-1)^(1/2))/(R
  ^4-1)^(1/2);
```

$$\frac{2\,\pi\,(-1-4\,R^8-3\,R^2\sqrt{R^4-1}+5\,R^4+4\,R^6\sqrt{R^4-1})}{\sqrt{R^4-1}}$$

Intégral(A1*B2)/2*Pi :

```
> restart;
```

```
> (1/(2*Pi))*(-b[1,1]/(4*R^3))*(-b[2,0]*(1/4*Pi-2*Pi/(-1+R^4)^(1/2
  )*R^6+2*Pi/(-1+R^4)^(1/2)*R^10+R^4*Pi-2*Pi*R^8)+(a[1,1]-b[0,2])*
  (3/4*Pi+2*Pi*R^8-2*Pi/(-1+R^4)^(1/2)*R^10-2*Pi/(-1+R^4)^(1/2)*R^
  2+4*Pi/(-1+R^4)^(1/2)*R^6-3*R^4*Pi)+(a[1,1]+b[2,0])*int(cos(phi)
  ^2*sin(phi)^2,phi=0..2*Pi)+b[0,2]*int(sin(phi)^4,phi=0..2*Pi))+(
  1/(2*Pi))*((b[2,0]*b[1,1]/(2*R))*(2*R^2*Pi*(-3*R^2+4*R^6+(R^4-1)
  ^(1/2)-4*R^4*(R^4-1)^(1/2))/(R^4-1)^(1/2))-((b[1,1]*(a[1,1]-b[0,
  2])/(2*R))*(2*Pi*(-1-4*R^8-3*R^2*(R^4-1)^(1/2)+5*R^4+4*R^6*(R^4-
  1)^(1/2))/(R^4-1)^(1/2))));
```

$$-\frac{1}{8}\,b_{1,1}\left(-b_{2,0}\left(\frac{\pi}{4}-\frac{2\,\pi\,R^6}{\sqrt{-1+R^4}}+\frac{2\,\pi\,R^{10}}{\sqrt{-1+R^4}}+R^4\,\pi-2\,\pi\,R^8\right)\right.$$
$$+(a_{1,1}-b_{0,2})\left(\frac{3\,\pi}{4}+2\,\pi\,R^8-\frac{2\,\pi\,R^{10}}{\sqrt{-1+R^4}}-\frac{2\,\pi\,R^2}{\sqrt{-1+R^4}}+\frac{4\,\pi\,R^6}{\sqrt{-1+R^4}}-3\,R^4\,\pi\right)$$
$$\left.+\frac{1}{4}(a_{1,1}+b_{2,0})\,\pi+\frac{3}{4}\,b_{0,2}\,\pi\right)\Big/(\pi\,R^3)+\frac{1}{2}\left(\right.$$
$$\frac{b_{2,0}\,b_{1,1}\,R\,\pi\,(-3\,R^2+4\,R^6+\sqrt{-1+R^4}-4\,R^4\sqrt{-1+R^4})}{\sqrt{-1+R^4}}$$

$$-\frac{b_{1,1}\,(a_{1,1}-b_{0,2})\,\pi\,(-1-4\,R^8-3\,R^2\sqrt{-1+R^4}+5\,R^4+4\,R^6\sqrt{-1+R^4})}{R\sqrt{-1+R^4}}\Bigg)/\pi$$

```
> expand(%);
```

$$-\frac{15}{8}b_{1,1}\,R\,b_{0,2}+\frac{9}{4}b_{1,1}\,R^5\,b_{0,2}-\frac{1}{8}\frac{b_{1,1}\,a_{1,1}}{R^3}-\frac{9}{4}b_{1,1}\,R^5\,b_{2,0}+\frac{5}{8}b_{1,1}\,R\,b_{2,0}+\frac{15}{8}b_{1,1}\,R\,a_{1,1}$$
$$-\frac{9}{4}b_{1,1}\,R^5\,a_{1,1}-\frac{7}{4}\frac{b_{1,1}\,R^3\,b_{2,0}}{\sqrt{-1+R^4}}+\frac{9}{4}\frac{b_{1,1}\,R^7\,b_{2,0}}{\sqrt{-1+R^4}}+\frac{9}{4}\frac{b_{1,1}\,R^7\,a_{1,1}}{\sqrt{-1+R^4}}+\frac{3}{4}\frac{b_{1,1}\,a_{1,1}}{R\sqrt{-1+R^4}}$$
$$-\frac{3\,b_{1,1}\,R^3\,a_{1,1}}{\sqrt{-1+R^4}}-\frac{3}{4}\frac{b_{1,1}\,b_{0,2}}{R\sqrt{-1+R^4}}+\frac{3\,b_{1,1}\,R^3\,b_{0,2}}{\sqrt{-1+R^4}}-\frac{9}{4}\frac{b_{1,1}\,R^7\,b_{0,2}}{\sqrt{-1+R^4}}$$

```
> normal(%);
```

$$-\frac{1}{8}b_{1,1}\,(15\,R^4\,b_{0,2}\sqrt{-1+R^4}-18\,R^8\,b_{0,2}\sqrt{-1+R^4}+a_{1,1}\sqrt{-1+R^4}+18\,R^8\,b_{2,0}\sqrt{-1+R^4}$$
$$-5\,R^4\,b_{2,0}\sqrt{-1+R^4}-15\,R^4\,a_{1,1}\sqrt{-1+R^4}+18\,R^8\,a_{1,1}\sqrt{-1+R^4}+14\,R^6\,b_{2,0}-18\,R^{10}\,b_{2,0}$$
$$-18\,R^{10}\,a_{1,1}-6\,a_{1,1}\,R^2+24\,R^6\,a_{1,1}+6\,b_{0,2}\,R^2-24\,R^6\,b_{0,2}+18\,R^{10}\,b_{0,2})\Big/(R^3\sqrt{-1+R^4})$$

```
> collect(%,R);
```

$$-\frac{1}{8}\frac{b_{1,1}\,(-18\,b_{2,0}-18\,a_{1,1}+18\,b_{0,2})\,R^7}{\sqrt{-1+R^4}}$$
$$-\frac{1}{8}\frac{b_{1,1}\,(-18\,b_{0,2}\sqrt{-1+R^4}+18\,a_{1,1}\sqrt{-1+R^4}+18\,b_{2,0}\sqrt{-1+R^4})\,R^5}{\sqrt{-1+R^4}}$$
$$-\frac{1}{8}\frac{b_{1,1}\,(14\,b_{2,0}-24\,b_{0,2}+24\,a_{1,1})\,R^3}{\sqrt{-1+R^4}}$$
$$-\frac{1}{8}\frac{b_{1,1}\,(-15\,a_{1,1}\sqrt{-1+R^4}+15\,b_{0,2}\sqrt{-1+R^4}-5\,b_{2,0}\sqrt{-1+R^4})\,R}{\sqrt{-1+R^4}}$$
$$-\frac{1}{8}\frac{b_{1,1}\,(6\,b_{0,2}-6\,a_{1,1})}{\sqrt{-1+R^4}\,R}-\frac{1}{8}\frac{b_{1,1}\,a_{1,1}}{R^3}$$

Intégral(A2*B1)/2*Pi :

```
> restart;
```

```
> (1/(2*Pi))*(((-b[1,1]*(b[2,0]+a[1,1]-b[0,2])*R/4+b[1,1]*a[1,1]/(
  4*R^3))*cos(phi)^2)+(b[1,1]*(b[2,0]+a[1,1]-b[0,2])*R^3/4-b[1,1]*
  (a[1,1]-b[0,2])/(4*R))*cos(phi)*ln(R^2+cos(phi)));
```

$$\frac{1}{2}\Bigg(\left(\left(-\frac{1}{4}b_{1,1}\,(b_{2,0}+a_{1,1}-b_{0,2})\,R+\frac{1}{4}\frac{b_{1,1}\,a_{1,1}}{R^3}\right)\cos(\phi)^2\right.$$
$$\left.+\left(\frac{1}{4}b_{1,1}\,(b_{2,0}+a_{1,1}-b_{0,2})\,R^3-\frac{1}{4}\frac{b_{1,1}\,(a_{1,1}-b_{0,2})}{R}\right)\cos(\phi)\,\ln(R^2+\cos(\phi))\right)/\pi$$

```
> Int((1/(2*Pi))*((-b[1,1]*(b[2,0]+a[1,1]-b[0,2])*R/4+b[1,1]*a[1,1]/(4*R^3))*cos(phi)^2),phi=0..2*Pi)=int((1/(2*Pi))*((-b[1,1]*(b[2,0]+a[1,1]-b[0,2])*R/4+b[1,1]*a[1,1]/(4*R^3))*cos(phi)^2),phi=0..2*Pi);
```

$$\int_0^{2\pi} \frac{1}{2}\frac{\left(-\frac{1}{4}b_{1,1}\left(b_{2,0}+a_{1,1}-b_{0,2}\right)R+\frac{1}{4}\frac{b_{1,1}a_{1,1}}{R^3}\right)\cos(\phi)^2}{\pi}\,d\phi =$$

$$-\frac{1}{8}\frac{b_{1,1}\left(R^4 b_{2,0}+R^4 a_{1,1}-R^4 b_{0,2}-a_{1,1}\right)}{R^3}$$

```
> (1/(2*Pi))*(1/4*b[1,1]*(b[2,0]+a[1,1]-b[0,2])*R^3-1/4*b[1,1]*(a[1,1]-b[0,2])/R)*2*Pi*(R^2-sqrt(R^4-1));
```

$$\left(\frac{1}{4}b_{1,1}\left(b_{2,0}+a_{1,1}-b_{0,2}\right)R^3-\frac{1}{4}\frac{b_{1,1}\left(a_{1,1}-b_{0,2}\right)}{R}\right)\left(R^2-\sqrt{R^4-1}\right)$$

```
> %-1/8*b[1,1]*(R^4*b[2,0]+R^4*a[1,1]-R^4*b[0,2]-a[1,1])/R^3;
```

$$\left(\frac{1}{4}b_{1,1}\left(b_{2,0}+a_{1,1}-b_{0,2}\right)R^3-\frac{1}{4}\frac{b_{1,1}\left(a_{1,1}-b_{0,2}\right)}{R}\right)\left(R^2-\sqrt{R^4-1}\right)$$

$$-\frac{1}{8}\frac{b_{1,1}\left(R^4 b_{2,0}+R^4 a_{1,1}-R^4 b_{0,2}-a_{1,1}\right)}{R^3}$$

```
> expand(%);
```

$$\frac{1}{4}b_{1,1}R^5 b_{2,0}-\frac{1}{4}b_{1,1}R^3 b_{2,0}\sqrt{R^4-1}+\frac{1}{4}b_{1,1}R^5 a_{1,1}-\frac{1}{4}b_{1,1}R^3 a_{1,1}\sqrt{R^4-1}-\frac{1}{4}b_{1,1}R^5 b_{0,2}$$

$$+\frac{1}{4}b_{1,1}R^3 b_{0,2}\sqrt{R^4-1}-\frac{3}{8}b_{1,1}R\,a_{1,1}+\frac{1}{4}\frac{b_{1,1}a_{1,1}\sqrt{R^4-1}}{R}+\frac{3}{8}b_{1,1}R\,b_{0,2}$$

$$-\frac{1}{4}\frac{b_{1,1}b_{0,2}\sqrt{R^4-1}}{R}-\frac{1}{8}b_{1,1}R\,b_{2,0}+\frac{1}{8}\frac{b_{1,1}a_{1,1}}{R^3}$$

```
> normal(%);
```

$$-\frac{1}{8}b_{1,1}\left(-2R^8 b_{2,0}+2R^6 b_{2,0}\sqrt{R^4-1}-2R^8 a_{1,1}+2R^6 a_{1,1}\sqrt{R^4-1}+2R^8 b_{0,2}\right.$$

$$\left.-2R^6 b_{0,2}\sqrt{R^4-1}+3R^4 a_{1,1}-2a_{1,1}\sqrt{R^4-1}\,R^2-3R^4 b_{0,2}+2b_{0,2}\sqrt{R^4-1}\,R^2+R^4 b_{2,0}\right.$$

$$\left.-a_{1,1}\right)\Big/R^3$$

```
> collect(%,R);
```

$$-\frac{1}{8}b_{1,1}\left(-2b_{2,0}+2b_{0,2}-2a_{1,1}\right)R^5$$

$$-\frac{1}{8}b_{1,1}\left(2b_{2,0}\sqrt{R^4-1}-2b_{0,2}\sqrt{R^4-1}+2a_{1,1}\sqrt{R^4-1}\right)R^3$$

$$-\frac{1}{8}b_{1,1}(-3\,b_{0,2}+3\,a_{1,1}+b_{2,0})\,R-\frac{1}{8}\frac{b_{1,1}(2\,b_{0,2}\sqrt{R^4-1}-2\,a_{1,1}\sqrt{R^4-1})}{R}+\frac{1}{8}\frac{b_{1,1}\,a_{1,1}}{R^3}$$

```
> F21:=-1/8*b[1,1]*(15*R^4*b[0,2]*(-1+R^4)^(1/2)-18*R^8*b[0,2]*(-1
  +R^4)^(1/2)+a[1,1]*(-1+R^4)^(1/2)+18*R^8*b[2,0]*(-1+R^4)^(1/2)-5
  *R^4*b[2,0]*(-1+R^4)^(1/2)-15*R^4*a[1,1]*(-1+R^4)^(1/2)+18*R^8*a
  [1,1]*(-1+R^4)^(1/2)+14*R^6*b[2,0]-18*R^10*b[2,0]-18*R^10*a[1,1]
  -6*a[1,1]*R^2+24*R^6*a[1,1]+6*b[0,2]*R^2-24*R^6*b[0,2]+18*R^10*b
  [0,2])/R^3/(-1+R^4)^(1/2)+%;
```

$$F21:=-\frac{1}{8}b_{1,1}(15\,R^4\,b_{0,2}\sqrt{R^4-1}-18\,R^8\,b_{0,2}\sqrt{R^4-1}+a_{1,1}\sqrt{R^4-1}+18\,R^8\,b_{2,0}\sqrt{R^4-1}$$
$$-5\,R^4\,b_{2,0}\sqrt{R^4-1}-15\,R^4\,a_{1,1}\sqrt{R^4-1}+18\,R^8\,a_{1,1}\sqrt{R^4-1}+14\,R^6\,b_{2,0}-18\,R^{10}\,b_{2,0}$$
$$-18\,R^{10}\,a_{1,1}-6\,a_{1,1}\,R^2+24\,R^6\,a_{1,1}+6\,b_{0,2}\,R^2-24\,R^6\,b_{0,2}+18\,R^{10}\,b_{0,2})\Big/(R^3\sqrt{R^4-1})$$
$$-\frac{1}{8}b_{1,1}(-2\,b_{2,0}+2\,b_{0,2}-2\,a_{1,1})\,R^5$$
$$-\frac{1}{8}b_{1,1}(2\,b_{2,0}\sqrt{R^4-1}-2\,b_{0,2}\sqrt{R^4-1}+2\,a_{1,1}\sqrt{R^4-1})\,R^3$$
$$-\frac{1}{8}b_{1,1}(-3\,b_{0,2}+3\,a_{1,1}+b_{2,0})\,R-\frac{1}{8}\frac{b_{1,1}(2\,b_{0,2}\sqrt{R^4-1}-2\,a_{1,1}\sqrt{R^4-1})}{R}+\frac{1}{8}\frac{b_{1,1}\,a_{1,1}}{R^3}$$

```
> expand(%);
```

$$-\frac{3}{2}b_{1,1}\,R\,b_{0,2}+\frac{3}{2}b_{1,1}\,R\,a_{1,1}-2\,b_{1,1}\,R^5\,b_{2,0}+2\,b_{1,1}\,R^5\,b_{0,2}-\frac{1}{4}b_{1,1}\,R^3\,a_{1,1}\sqrt{R^4-1}$$
$$+\frac{1}{2}b_{1,1}\,R\,b_{2,0}-\frac{1}{4}\frac{b_{1,1}\,b_{0,2}\sqrt{R^4-1}}{R}-\frac{1}{4}b_{1,1}\,R^3\,b_{2,0}\sqrt{R^4-1}-2\,b_{1,1}\,R^5\,a_{1,1}$$
$$+\frac{1}{4}\frac{b_{1,1}\,a_{1,1}\sqrt{R^4-1}}{R}+\frac{1}{4}b_{1,1}\,R^3\,b_{0,2}\sqrt{R^4-1}-\frac{7}{4}\frac{b_{1,1}\,R^3\,b_{2,0}}{\sqrt{R^4-1}}+\frac{9}{4}\frac{b_{1,1}\,R^7\,b_{2,0}}{\sqrt{R^4-1}}$$
$$+\frac{9}{4}\frac{b_{1,1}\,R^7\,a_{1,1}}{\sqrt{R^4-1}}+\frac{3}{4}\frac{b_{1,1}\,a_{1,1}}{R\sqrt{R^4-1}}-\frac{3\,b_{1,1}\,R^3\,a_{1,1}}{\sqrt{R^4-1}}-\frac{3}{4}\frac{b_{1,1}\,b_{0,2}}{R\sqrt{R^4-1}}+\frac{3\,b_{1,1}\,R^3\,b_{0,2}}{\sqrt{R^4-1}}$$
$$-\frac{9}{4}\frac{b_{1,1}\,R^7\,b_{0,2}}{\sqrt{R^4-1}}$$

```
> simplify(collect(%,R));
```

$$-\frac{1}{2}b_{1,1}(-4\,R^8\,b_{2,0}-4\,R^8\,a_{1,1}+4\,R^8\,b_{0,2}+4\,R^6\,b_{2,0}\sqrt{R^4-1}+4\,R^6\,a_{1,1}\sqrt{R^4-1}$$
$$-4\,R^6\,b_{0,2}\sqrt{R^4-1}+5\,R^4\,a_{1,1}+3\,R^4\,b_{2,0}-5\,R^4\,b_{0,2}+3\,b_{0,2}\sqrt{R^4-1}\,R^2$$
$$-3\,a_{1,1}\sqrt{R^4-1}\,R^2-R^2\sqrt{R^4-1}\,b_{2,0}-a_{1,1}+b_{0,2})\Big/(\sqrt{R^4-1}\,R)$$

```
> expand(%);
```

$$\frac{2\,b_{1,1}\,R^7\,b_{2,0}}{\sqrt{R^4-1}}+\frac{2\,b_{1,1}\,R^7\,a_{1,1}}{\sqrt{R^4-1}}-\frac{2\,b_{1,1}\,R^7\,b_{0,2}}{\sqrt{R^4-1}}-2\,b_{1,1}\,R^5\,b_{2,0}-2\,b_{1,1}\,R^5\,a_{1,1}+2\,b_{1,1}\,R^5\,b_{0,2}$$
$$-\frac{5}{2}\frac{b_{1,1}\,R^3\,a_{1,1}}{\sqrt{R^4-1}}-\frac{3}{2}\frac{b_{1,1}\,R^3\,b_{2,0}}{\sqrt{R^4-1}}+\frac{5}{2}\frac{b_{1,1}\,R^3\,b_{0,2}}{\sqrt{R^4-1}}-\frac{3}{2}\,b_{1,1}\,R\,b_{0,2}+\frac{3}{2}\,b_{1,1}\,R\,a_{1,1}+\frac{1}{2}\,b_{1,1}\,R\,b_{2,0}$$
$$+\frac{1}{2}\frac{b_{1,1}\,a_{1,1}}{R\sqrt{R^4-1}}-\frac{1}{2}\frac{b_{1,1}\,b_{0,2}}{R\sqrt{R^4-1}}$$

```
> collect(%,R);
```

$$\left(\frac{2\,b_{1,1}\,a_{1,1}}{\sqrt{R^4-1}}-\frac{2\,b_{1,1}\,b_{0,2}}{\sqrt{R^4-1}}+\frac{2\,b_{1,1}\,b_{2,0}}{\sqrt{R^4-1}}\right)R^7+(-2\,b_{1,1}\,a_{1,1}+2\,b_{1,1}\,b_{0,2}-2\,b_{1,1}\,b_{2,0})\,R^5$$
$$+\left(-\frac{5}{2}\frac{b_{1,1}\,a_{1,1}}{\sqrt{R^4-1}}-\frac{3}{2}\frac{b_{1,1}\,b_{2,0}}{\sqrt{R^4-1}}+\frac{5}{2}\frac{b_{1,1}\,b_{0,2}}{\sqrt{R^4-1}}\right)R^3+\left(-\frac{3}{2}\,b_{1,1}\,b_{0,2}+\frac{3}{2}\,b_{1,1}\,a_{1,1}+\frac{1}{2}\,b_{1,1}\,b_{2,0}\right)R$$
$$+\frac{-\frac{1}{2}\frac{b_{1,1}\,b_{0,2}}{\sqrt{R^4-1}}+\frac{1}{2}\frac{b_{1,1}\,a_{1,1}}{\sqrt{R^4-1}}}{R}$$

F22 :

```
> restart;
> P:=-y+x*y;
```

$$P := -y + x\,y$$

```
> Q:=x+y^2;
```

$$Q := x + y^2$$

```
> Q*(a[2,0]*x^2+a[1,1]*x*y+a[0,2]*y^2)-P*(b[2,0]*x^2+b[1,1]*x*y+b[
  0,2]*y^2);
```

$$(x+y^2)\,(a_{2,0}\,x^2+a_{1,1}\,x\,y+a_{0,2}\,y^2)-(-y+x\,y)\,(b_{2,0}\,x^2+b_{1,1}\,x\,y+b_{0,2}\,y^2)$$

```
> H:=simplify(%);
```

$$H := a_{2,0}\,x^3+a_{1,1}\,x^2\,y+x\,a_{0,2}\,y^2+y^2\,a_{2,0}\,x^2+y^3\,a_{1,1}\,x+y^4\,a_{0,2}+y\,b_{2,0}\,x^2+b_{1,1}\,x\,y^2+b_{0,2}\,y^3$$
$$-x^3\,y\,b_{2,0}-x^2\,y^2\,b_{1,1}-x\,y^3\,b_{0,2}$$

```
> x*(b[2,0]*x^2+b[1,1]*x*y+b[0,2]*y^2)-y*(a[2,0]*x^2+a[1,1]*x*y+a[
  0,2]*y^2);
```

$$x\,(b_{2,0}\,x^2+b_{1,1}\,x\,y+b_{0,2}\,y^2)-y\,(a_{2,0}\,x^2+a_{1,1}\,x\,y+a_{0,2}\,y^2)$$

```
> expand(%*H);
```

$$-{a_{0,2}}^2\,y^5\,x-y^3\,{a_{2,0}}^2\,x^4-{a_{0,2}}^2\,y^7+b_{2,0}\,x^5\,a_{1,1}\,y+b_{2,0}\,x^4\,a_{0,2}\,y^2+2\,b_{2,0}\,x^5\,y^2\,a_{2,0}$$
$$+2\,b_{2,0}\,x^4\,y^3\,a_{1,1}+2\,b_{2,0}\,x^3\,y^4\,a_{0,2}+2\,b_{2,0}\,x^4\,b_{1,1}\,y^2+2\,b_{2,0}\,x^3\,b_{0,2}\,y^3-2\,b_{2,0}\,x^5\,y^2\,b_{1,1}$$
$$-2\,b_{2,0}\,x^4\,y^3\,b_{0,2}+b_{1,1}\,x^5\,y\,a_{2,0}+b_{1,1}\,x^4\,y^2\,a_{1,1}+b_{1,1}\,x^3\,y^3\,a_{0,2}+2\,b_{1,1}\,x^4\,y^3\,a_{2,0}$$
$$+2\,b_{1,1}\,x^3\,y^4\,a_{1,1}+2\,b_{1,1}\,x^2\,y^5\,a_{0,2}+2\,b_{1,1}\,x^2\,y^4\,b_{0,2}-2\,b_{1,1}\,x^3\,y^4\,b_{0,2}+x^4\,b_{0,2}\,y^2\,a_{2,0}$$
$$+x^3\,b_{0,2}\,y^3\,a_{1,1}+x^2\,b_{0,2}\,y^4\,a_{0,2}+2\,x^3\,b_{0,2}\,y^4\,a_{2,0}+2\,x^2\,b_{0,2}\,y^5\,a_{1,1}+2\,x\,b_{0,2}\,y^6\,a_{0,2}$$

$$-2\,y^2\,a_{2,0}\,x^4\,a_{1,1}-2\,y^3\,a_{2,0}\,x^3\,a_{0,2}-2\,y^4\,a_{2,0}\,x^3\,a_{1,1}-2\,y^5\,a_{2,0}\,x^2\,a_{0,2}-y^2\,a_{2,0}\,x^4\,b_{2,0}$$
$$-y^3\,a_{2,0}\,x^3\,b_{1,1}-y^4\,a_{2,0}\,x^2\,b_{0,2}-2\,a_{1,1}\,x^2\,y^4\,a_{0,2}-2\,a_{1,1}\,x\,y^6\,a_{0,2}-a_{1,1}\,x^3\,y^3\,b_{2,0}$$
$$-a_{1,1}\,x^2\,y^4\,b_{1,1}-a_{1,1}\,x\,y^5\,b_{0,2}-a_{0,2}\,y^4\,b_{2,0}\,x^2-a_{0,2}\,y^5\,b_{1,1}\,x-a_{1,1}^{\,2}\,x^2\,y^5-a_{1,1}^{\,2}\,x^3\,y^3$$
$$-y\,a_{2,0}^{\,2}\,x^5-x^2\,b_{0,2}^{\,2}\,y^5+x\,b_{0,2}^{\,2}\,y^5-b_{1,1}^{\,2}\,x^4\,y^3-b_{2,0}^{\,2}\,x^6\,y+b_{1,1}^{\,2}\,x^3\,y^3+b_{2,0}^{\,2}\,x^5\,y$$
$$+b_{2,0}\,x^6\,a_{2,0}-a_{0,2}\,y^6\,b_{0,2}$$

```
> subs(a[0,2]=0,a[2,0]=b[1,1],%);
```

$$b_{2,0}\,x^5\,a_{1,1}\,y+2\,b_{2,0}\,x^4\,y^3\,a_{1,1}+b_{2,0}\,x^4\,b_{1,1}\,y^2+2\,b_{2,0}\,x^3\,b_{0,2}\,y^3-2\,b_{2,0}\,x^4\,y^3\,b_{0,2}$$
$$-b_{1,1}\,x^4\,y^2\,a_{1,1}+b_{1,1}\,x^2\,y^4\,b_{0,2}+x^3\,b_{0,2}\,y^3\,a_{1,1}+2\,x^2\,b_{0,2}\,y^5\,a_{1,1}-a_{1,1}\,x^3\,y^3\,b_{2,0}$$
$$-a_{1,1}\,x^2\,y^4\,b_{1,1}-a_{1,1}\,x\,y^5\,b_{0,2}-a_{1,1}^{\,2}\,x^2\,y^5-a_{1,1}^{\,2}\,x^3\,y^3-x^2\,b_{0,2}^{\,2}\,y^5+x\,b_{0,2}^{\,2}\,y^5-b_{2,0}^{\,2}\,x^6\,y$$
$$+b_{2,0}^{\,2}\,x^5\,y+x^4\,b_{0,2}\,y^2\,b_{1,1}+b_{2,0}\,x^6\,b_{1,1}$$

```
> sort(%,y);
```

$$2\,x^2\,b_{0,2}\,a_{1,1}\,y^5-x^2\,b_{0,2}^{\,2}\,y^5+x\,b_{0,2}^{\,2}\,y^5-a_{1,1}\,x\,b_{0,2}\,y^5-a_{1,1}^{\,2}\,x^2\,y^5+b_{1,1}\,x^2\,b_{0,2}\,y^4$$
$$-a_{1,1}\,x^2\,b_{1,1}\,y^4-a_{1,1}\,x^3\,b_{2,0}\,y^3+2\,b_{2,0}\,x^3\,b_{0,2}\,y^3-a_{1,1}^{\,2}\,x^3\,y^3+x^3\,b_{0,2}\,a_{1,1}\,y^3$$
$$-2\,b_{2,0}\,x^4\,b_{0,2}\,y^3+2\,b_{2,0}\,x^4\,a_{1,1}\,y^3-b_{1,1}\,x^4\,a_{1,1}\,y^2+b_{2,0}\,x^4\,b_{1,1}\,y^2+x^4\,b_{0,2}\,b_{1,1}\,y^2$$
$$+b_{2,0}\,x^5\,a_{1,1}\,y-b_{2,0}^{\,2}\,x^6\,y+b_{2,0}^{\,2}\,x^5\,y+b_{2,0}\,x^6\,b_{1,1}$$

```
> simplify(subs(x=rho*cos(phi),y=rho*sin(phi),%/(2*R*(x^2+y^2)^(5/
  2))));
```

$$-\frac{1}{2}\,\mathrm{csgn}(\rho)\,\rho\,\cos(\phi)\,(2\,\sin(\phi)\,\cos(\phi)^4\,b_{2,0}\,b_{0,2}+\sin(\phi)\,\cos(\phi)\,\rho\,a_{1,1}^{\,2}$$
$$+\sin(\phi)\,\cos(\phi)\,\rho\,b_{0,2}^{\,2}+2\,\sin(\phi)\,\cos(\phi)^4\,a_{1,1}\,b_{0,2}-2\,\sin(\phi)\,\cos(\phi)^3\,\rho\,a_{1,1}^{\,2}$$
$$-2\,\sin(\phi)\,\cos(\phi)^3\,\rho\,b_{0,2}^{\,2}-3\,\sin(\phi)\,\cos(\phi)^2\,a_{1,1}\,b_{0,2}+\sin(\phi)\,\cos(\phi)^5\,\rho\,a_{1,1}^{\,2}$$
$$+\sin(\phi)\,\cos(\phi)^2\,b_{2,0}\,a_{1,1}-2\,\sin(\phi)\,\cos(\phi)^2\,b_{2,0}\,b_{0,2}+\sin(\phi)\,\cos(\phi)^5\,\rho\,b_{0,2}^{\,2}-\sin(\phi)\,b_{0,2}^{\,2}$$
$$-\sin(\phi)\,\cos(\phi)^4\,b_{0,2}^{\,2}-\cos(\phi)^3\,a_{1,1}\,b_{1,1}+\cos(\phi)^3\,b_{1,1}\,b_{0,2}+\cos(\phi)\,a_{1,1}\,b_{1,1}$$
$$-\cos(\phi)\,b_{1,1}\,b_{0,2}-\cos(\phi)^3\,b_{1,1}\,b_{2,0}+\sin(\phi)\,\cos(\phi)^2\,a_{1,1}^{\,2}+2\,\sin(\phi)\,\cos(\phi)^2\,b_{0,2}^{\,2}$$
$$+\sin(\phi)\,a_{1,1}\,b_{0,2}-\sin(\phi)\,\cos(\phi)^4\,a_{1,1}^{\,2}-2\,\sin(\phi)\,\cos(\phi)\,\rho\,b_{0,2}\,a_{1,1}$$
$$+4\,\sin(\phi)\,\cos(\phi)^3\,\rho\,b_{0,2}\,a_{1,1}+2\,\sin(\phi)\,\cos(\phi)^5\,\rho\,b_{2,0}\,a_{1,1}-2\,\sin(\phi)\,\cos(\phi)^5\,\rho\,b_{2,0}\,b_{0,2}$$
$$-2\,\sin(\phi)\,\cos(\phi)^5\,\rho\,b_{0,2}\,a_{1,1}-2\,\sin(\phi)\,\cos(\phi)^3\,\rho\,b_{2,0}\,a_{1,1}+2\,\sin(\phi)\,\cos(\phi)^3\,\rho\,b_{2,0}\,b_{0,2}$$
$$-2\,b_{2,0}\,\cos(\phi)^4\,a_{1,1}\,\sin(\phi)+b_{2,0}^{\,2}\,\rho\,\cos(\phi)^5\,\sin(\phi)-b_{2,0}^{\,2}\,\cos(\phi)^4\,\sin(\phi))/R$$

```
> subs(rho=1/(R^2+cos(phi)),%);
```

$$-\frac{1}{2}\,\mathrm{csgn}\left(\frac{1}{R^2+\cos(\phi)}\right)\cos(\phi)\left(2\,\sin(\phi)\,\cos(\phi)^4\,b_{2,0}\,b_{0,2}+2\,\sin(\phi)\,\cos(\phi)^4\,a_{1,1}\,b_{0,2}\right.$$

$$
\begin{aligned}
&- 3\sin(\phi)\cos(\phi)^2 a_{1,1} b_{0,2} + \sin(\phi)\cos(\phi)^2 b_{2,0} a_{1,1} - 2\sin(\phi)\cos(\phi)^2 b_{2,0} b_{0,2} \\
&- \sin(\phi) {b_{0,2}}^2 - \sin(\phi)\cos(\phi)^4 {b_{0,2}}^2 - \cos(\phi)^3 a_{1,1} b_{1,1} + \cos(\phi)^3 b_{1,1} b_{0,2} + \cos(\phi) a_{1,1} b_{1,1} \\
&- \cos(\phi) b_{1,1} b_{0,2} - \cos(\phi)^3 b_{1,1} b_{2,0} + \sin(\phi)\cos(\phi)^2 {a_{1,1}}^2 + 2\sin(\phi)\cos(\phi)^2 {b_{0,2}}^2 \\
&+ \sin(\phi) a_{1,1} b_{0,2} - \sin(\phi)\cos(\phi)^4 {a_{1,1}}^2 + \frac{\sin(\phi)\cos(\phi) {a_{1,1}}^2}{R^2+\cos(\phi)} + \frac{\sin(\phi)\cos(\phi) {b_{0,2}}^2}{R^2+\cos(\phi)} \\
&- \frac{2\sin(\phi)\cos(\phi)^3 {a_{1,1}}^2}{R^2+\cos(\phi)} - \frac{2\sin(\phi)\cos(\phi)^3 {b_{0,2}}^2}{R^2+\cos(\phi)} + \frac{\sin(\phi)\cos(\phi)^5 {a_{1,1}}^2}{R^2+\cos(\phi)} \\
&+ \frac{\sin(\phi)\cos(\phi)^5 {b_{0,2}}^2}{R^2+\cos(\phi)} + \frac{{b_{2,0}}^2 \cos(\phi)^5 \sin(\phi)}{R^2+\cos(\phi)} - \frac{2\sin(\phi)\cos(\phi) b_{0,2} a_{1,1}}{R^2+\cos(\phi)} \\
&+ \frac{4\sin(\phi)\cos(\phi)^3 b_{0,2} a_{1,1}}{R^2+\cos(\phi)} + \frac{2\sin(\phi)\cos(\phi)^5 b_{2,0} a_{1,1}}{R^2+\cos(\phi)} - \frac{2\sin(\phi)\cos(\phi)^5 b_{2,0} b_{0,2}}{R^2+\cos(\phi)} \\
&- \frac{2\sin(\phi)\cos(\phi)^5 b_{0,2} a_{1,1}}{R^2+\cos(\phi)} - \frac{2\sin(\phi)\cos(\phi)^3 b_{2,0} a_{1,1}}{R^2+\cos(\phi)} + \frac{2\sin(\phi)\cos(\phi)^3 b_{2,0} b_{0,2}}{R^2+\cos(\phi)} \\
&- 2 b_{2,0}\cos(\phi)^4 a_{1,1}\sin(\phi) - {b_{2,0}}^2\cos(\phi)^4\sin(\phi)\Big) \Big/ \left(\left(R^2+\cos(\phi)\right) R\right)
\end{aligned}
$$

```
> -1/2*cos(phi)*(-cos(phi)^3*a[1,1]*b[1,1]+cos(phi)^3*b[1,1]*b[0,2
  ]-cos(phi)^3*b[1,1]*b[2,0]-cos(phi)*b[1,1]*b[0,2]+cos(phi)*a[1,1
  ]*b[1,1])/(R*(R^2+cos(phi)));
```

$$
-\frac{1}{2}\cos(\phi)
$$

$$
\left(-\cos(\phi)^3 a_{1,1} b_{1,1} + \cos(\phi)^3 b_{1,1} b_{0,2} - \cos(\phi)^3 b_{1,1} b_{2,0} - \cos(\phi) b_{1,1} b_{0,2} + \cos(\phi) a_{1,1} b_{1,1}\right) \Big/ \left(R\left(R^2+\cos(\phi)\right)\right)
$$

```
> int((%)/(2*Pi),phi=0..2*Pi);
```

$$
\begin{aligned}
&-\frac{1}{4} b_{1,1} R \Bigg(2R^4 b_{2,0}\sqrt{R^4-1} - 2R^2 \operatorname{csgn}\left(\frac{R^2-1}{\sqrt{R^4-1}}\right) b_{0,2} + 2R^4 a_{1,1}\sqrt{R^4-1} \\
&+ 2R^2 \operatorname{csgn}\left(\frac{R^2-1}{\sqrt{R^4-1}}\right) a_{1,1} - 2R^6 \operatorname{csgn}\left(\frac{R^2-1}{\sqrt{R^4-1}}\right) b_{2,0} - 2R^6 \operatorname{csgn}\left(\frac{R^2-1}{\sqrt{R^4-1}}\right) a_{1,1} \\
&+ b_{0,2}\sqrt{R^4-1} - a_{1,1}\sqrt{R^4-1} + 2R^6 \operatorname{csgn}\left(\frac{R^2-1}{\sqrt{R^4-1}}\right) b_{0,2} - 2R^4 b_{0,2}\sqrt{R^4-1} \\
&+ b_{2,0}\sqrt{R^4-1}\Bigg) \Big/ \sqrt{R^4-1}
\end{aligned}
$$

```
> expand(1/4*b[1,1]*R*(-2*R^6*b[0,2]+2*R^2*b[0,2]+2*R^6*a[1,1]-2*R
  ^2*a[1,1]+2*R^6*b[2,0]+a[1,1]*(R^4-1)^(1/2)-b[2,0]*(R^4-1)^(1/2)
  -2*R^4*a[1,1]*(R^4-1)^(1/2)-2*R^4*b[2,0]*(R^4-1)^(1/2)-b[0,2]*(R
```

```
^4-1)^(1/2)+2*R^4*b[0,2]*(R^4-1)^(1/2))/(R^4-1)^(1/2));
```

$$-\frac{1}{2}\frac{b_{1,1}R^7 b_{0,2}}{\sqrt{R^4-1}}+\frac{1}{2}\frac{b_{1,1}R^3 b_{0,2}}{\sqrt{R^4-1}}+\frac{1}{2}\frac{b_{1,1}R^7 a_{1,1}}{\sqrt{R^4-1}}-\frac{1}{2}\frac{b_{1,1}R^3 a_{1,1}}{\sqrt{R^4-1}}+\frac{1}{2}\frac{b_{1,1}R^7 b_{2,0}}{\sqrt{R^4-1}}$$
$$+\frac{1}{4}b_{1,1}R\,a_{1,1}-\frac{1}{4}b_{1,1}R\,b_{2,0}-\frac{1}{2}b_{1,1}R^5 a_{1,1}-\frac{1}{2}b_{1,1}R^5 b_{2,0}-\frac{1}{4}b_{1,1}R\,b_{0,2}+\frac{1}{2}b_{1,1}R^5 b_{0,2}$$

```
> collect(%,R);
```

$$\left(-\frac{1}{2}\frac{b_{1,1}b_{0,2}}{\sqrt{R^4-1}}+\frac{1}{2}\frac{b_{1,1}b_{2,0}}{\sqrt{R^4-1}}+\frac{1}{2}\frac{b_{1,1}a_{1,1}}{\sqrt{R^4-1}}\right)R^7+\left(-\frac{1}{2}b_{1,1}b_{2,0}+\frac{1}{2}b_{1,1}b_{0,2}-\frac{1}{2}b_{1,1}a_{1,1}\right)R^5$$
$$+\left(-\frac{1}{2}\frac{b_{1,1}a_{1,1}}{\sqrt{R^4-1}}+\frac{1}{2}\frac{b_{1,1}b_{0,2}}{\sqrt{R^4-1}}\right)R^3+\left(\frac{1}{4}b_{1,1}a_{1,1}-\frac{1}{4}b_{1,1}b_{2,0}-\frac{1}{4}b_{1,1}b_{0,2}\right)R$$

```
> collect(%,b[1,1]);
```

$$\left(\left(-\frac{1}{2}\frac{b_{0,2}}{\sqrt{R^4-1}}+\frac{1}{2}\frac{b_{2,0}}{\sqrt{R^4-1}}+\frac{1}{2}\frac{a_{1,1}}{\sqrt{R^4-1}}\right)R^7+\left(-\frac{1}{2}b_{2,0}+\frac{1}{2}b_{0,2}-\frac{1}{2}a_{1,1}\right)R^5\right.$$
$$\left.+\left(-\frac{1}{2}\frac{a_{1,1}}{\sqrt{R^4-1}}+\frac{1}{2}\frac{b_{0,2}}{\sqrt{R^4-1}}\right)R^3+\left(\frac{1}{4}a_{1,1}-\frac{1}{4}b_{2,0}-\frac{1}{4}b_{0,2}\right)R\right)b_{1,1}$$

```
> expand(-1/4*(-2*R^6*A[2,0]*(R^4-1)^(1/2)-3*R^2*A[0,2]*(R^4-1)^(1
/2)-2*R^8*B[1,1]+2*R^6*B[1,1]*(R^4-1)^(1/2)-2*R^8*A[0,2]+R^2*A[2
,0]*(R^4-1)^(1/2)+2*R^4*B[1,1]-R^2*B[1,1]*(R^4-1)^(1/2)-2*R^4*A[
2,0]+2*R^6*A[0,2]*(R^4-1)^(1/2)-2*A[0,2]+2*R^8*A[2,0]+4*R^4*A[0,
2])/(R^4-1)^(1/2)/R)-%;
```

$$\frac{1}{2}R^5 A_{2,0}+\frac{3}{4}R\,A_{0,2}+\frac{1}{2}\frac{R^7 B_{1,1}}{\sqrt{R^4-1}}-\frac{1}{2}R^5 B_{1,1}+\frac{1}{2}\frac{R^7 A_{0,2}}{\sqrt{R^4-1}}-\frac{1}{4}R\,A_{2,0}-\frac{1}{2}\frac{R^3 B_{1,1}}{\sqrt{R^4-1}}+\frac{1}{4}R\,B_{1,1}$$
$$+\frac{1}{2}\frac{R^3 A_{2,0}}{\sqrt{R^4-1}}-\frac{1}{2}R^5 A_{0,2}+\frac{1}{2}\frac{A_{0,2}}{\sqrt{R^4-1}\,R}-\frac{1}{2}\frac{R^7 A_{2,0}}{\sqrt{R^4-1}}-\frac{R^3 A_{0,2}}{\sqrt{R^4-1}}-\left(\right.$$
$$\left(-\frac{1}{2}\frac{b_{0,2}}{\sqrt{R^4-1}}+\frac{1}{2}\frac{b_{2,0}}{\sqrt{R^4-1}}+\frac{1}{2}\frac{a_{1,1}}{\sqrt{R^4-1}}\right)R^7+\left(-\frac{1}{2}b_{2,0}+\frac{1}{2}b_{0,2}-\frac{1}{2}a_{1,1}\right)R^5$$
$$\left.+\left(-\frac{1}{2}\frac{a_{1,1}}{\sqrt{R^4-1}}+\frac{1}{2}\frac{b_{0,2}}{\sqrt{R^4-1}}\right)R^3+\left(\frac{1}{4}a_{1,1}-\frac{1}{4}b_{2,0}-\frac{1}{4}b_{0,2}\right)R\right)b_{1,1}$$

```
> expand(%);
```

$$\frac{1}{2}R^5 A_{2,0}+\frac{3}{4}R\,A_{0,2}+\frac{1}{2}\frac{R^7 B_{1,1}}{\sqrt{R^4-1}}-\frac{1}{2}R^5 B_{1,1}+\frac{1}{2}\frac{R^7 A_{0,2}}{\sqrt{R^4-1}}-\frac{1}{4}R\,A_{2,0}-\frac{1}{2}\frac{R^3 B_{1,1}}{\sqrt{R^4-1}}+\frac{1}{4}R\,B_{1,1}$$
$$+\frac{1}{2}\frac{R^3 A_{2,0}}{\sqrt{R^4-1}}-\frac{1}{2}R^5 A_{0,2}+\frac{1}{2}\frac{A_{0,2}}{\sqrt{R^4-1}\,R}-\frac{1}{2}\frac{R^7 A_{2,0}}{\sqrt{R^4-1}}-\frac{R^3 A_{0,2}}{\sqrt{R^4-1}}+\frac{1}{2}\frac{b_{1,1}R^7 b_{0,2}}{\sqrt{R^4-1}}$$
$$-\frac{1}{2}\frac{b_{1,1}R^3 b_{0,2}}{\sqrt{R^4-1}}-\frac{1}{2}\frac{b_{1,1}R^7 a_{1,1}}{\sqrt{R^4-1}}+\frac{1}{2}\frac{b_{1,1}R^3 a_{1,1}}{\sqrt{R^4-1}}-\frac{1}{2}\frac{b_{1,1}R^7 b_{2,0}}{\sqrt{R^4-1}}-\frac{1}{4}b_{1,1}R\,a_{1,1}$$

$$+\frac{1}{4}b_{1,1}\,R\,b_{2,0}+\frac{1}{2}b_{1,1}\,R^5\,a_{1,1}+\frac{1}{2}b_{1,1}\,R^5\,b_{2,0}+\frac{1}{4}b_{1,1}\,R\,b_{0,2}-\frac{1}{2}b_{1,1}\,R^5\,b_{0,2}$$

```
> F22:=collect(%,R);
```

$$F22 := \left(\frac{1}{2}\frac{b_{1,1}\,b_{0,2}}{\sqrt{R^4-1}}-\frac{1}{2}\frac{A_{2,0}}{\sqrt{R^4-1}}+\frac{1}{2}\frac{A_{0,2}}{\sqrt{R^4-1}}-\frac{1}{2}\frac{b_{1,1}\,b_{2,0}}{\sqrt{R^4-1}}+\frac{1}{2}\frac{B_{1,1}}{\sqrt{R^4-1}}-\frac{1}{2}\frac{b_{1,1}\,a_{1,1}}{\sqrt{R^4-1}}\right)R^7$$
$$+\left(\frac{1}{2}A_{2,0}-\frac{1}{2}B_{1,1}+\frac{1}{2}b_{1,1}\,b_{2,0}-\frac{1}{2}b_{1,1}\,b_{0,2}+\frac{1}{2}b_{1,1}\,a_{1,1}-\frac{1}{2}A_{0,2}\right)R^5$$
$$+\left(\frac{1}{2}\frac{A_{2,0}}{\sqrt{R^4-1}}-\frac{A_{0,2}}{\sqrt{R^4-1}}+\frac{1}{2}\frac{b_{1,1}\,a_{1,1}}{\sqrt{R^4-1}}-\frac{1}{2}\frac{B_{1,1}}{\sqrt{R^4-1}}-\frac{1}{2}\frac{b_{1,1}\,b_{0,2}}{\sqrt{R^4-1}}\right)R^3$$
$$+\left(\frac{3}{4}A_{0,2}+\frac{1}{4}B_{1,1}-\frac{1}{4}A_{2,0}+\frac{1}{4}b_{1,1}\,b_{2,0}+\frac{1}{4}b_{1,1}\,b_{0,2}-\frac{1}{4}b_{1,1}\,a_{1,1}\right)R+\frac{1}{2}\frac{A_{0,2}}{\sqrt{R^4-1}\,R}$$

```
> F21:=(2*b[1,1]/(R^4-1)^(1/2)*a[1,1]-2*b[1,1]/(R^4-1)^(1/2)*b[0,2
  ]+2*b[1,1]/(R^4-1)^(1/2)*b[2,0])*R^7+(-2*b[1,1]*a[1,1]+2*b[1,1]*
  b[0,2]-2*b[1,1]*b[2,0])*R^5+(-5/2*b[1,1]/(R^4-1)^(1/2)*a[1,1]-3/
  2*b[1,1]/(R^4-1)^(1/2)*b[2,0]+5/2*b[1,1]/(R^4-1)^(1/2)*b[0,2])*R
  ^3+(-3/2*b[1,1]*b[0,2]+3/2*b[1,1]*a[1,1]+1/2*b[1,1]*b[2,0])*R+(-
  1/2*b[1,1]/(R^4-1)^(1/2)*b[0,2]+1/2*b[1,1]/(R^4-1)^(1/2)*a[1,1])
  /R;
```

$$F21 := \left(\frac{2\,b_{1,1}\,a_{1,1}}{\sqrt{R^4-1}}-\frac{2\,b_{1,1}\,b_{0,2}}{\sqrt{R^4-1}}+\frac{2\,b_{1,1}\,b_{2,0}}{\sqrt{R^4-1}}\right)R^7$$
$$+\left(-2\,b_{1,1}\,a_{1,1}+2\,b_{1,1}\,b_{0,2}-2\,b_{1,1}\,b_{2,0}\right)R^5+\left(-\frac{5}{2}\frac{b_{1,1}\,a_{1,1}}{\sqrt{R^4-1}}-\frac{3}{2}\frac{b_{1,1}\,b_{2,0}}{\sqrt{R^4-1}}+\frac{5}{2}\frac{b_{1,1}\,b_{0,2}}{\sqrt{R^4-1}}\right)R^3$$
$$+\left(-\frac{3}{2}b_{1,1}\,b_{0,2}+\frac{3}{2}b_{1,1}\,a_{1,1}+\frac{1}{2}b_{1,1}\,b_{2,0}\right)R+\frac{-\frac{1}{2}\frac{b_{1,1}\,b_{0,2}}{\sqrt{R^4-1}}+\frac{1}{2}\frac{b_{1,1}\,a_{1,1}}{\sqrt{R^4-1}}}{R}$$

```
> expand(F22+F21);
```

$$\frac{1}{2}R^5\,A_{2,0}+\frac{1}{4}R\,B_{1,1}-\frac{1}{2}\frac{R^3\,B_{1,1}}{\sqrt{R^4-1}}-\frac{3}{2}\frac{R^3\,b_{1,1}\,b_{2,0}}{\sqrt{R^4-1}}-\frac{1}{2}\frac{b_{1,1}\,b_{0,2}}{R\sqrt{R^4-1}}+\frac{1}{2}\frac{b_{1,1}\,a_{1,1}}{R\sqrt{R^4-1}}$$
$$+\frac{1}{2}\frac{R^7\,B_{1,1}}{\sqrt{R^4-1}}+\frac{3}{4}R\,A_{0,2}-\frac{R^3\,A_{0,2}}{\sqrt{R^4-1}}+\frac{1}{2}\frac{R^3\,A_{2,0}}{\sqrt{R^4-1}}-\frac{1}{2}R^5\,A_{0,2}+\frac{3}{2}\frac{b_{1,1}\,R^7\,b_{2,0}}{\sqrt{R^4-1}}-\frac{2\,b_{1,1}\,R^3\,a_{1,1}}{\sqrt{R^4-1}}$$
$$+\frac{3}{2}\frac{b_{1,1}\,R^7\,a_{1,1}}{\sqrt{R^4-1}}+\frac{2\,b_{1,1}\,R^3\,b_{0,2}}{\sqrt{R^4-1}}-\frac{3}{2}\frac{b_{1,1}\,R^7\,b_{0,2}}{\sqrt{R^4-1}}+\frac{1}{2}\frac{R^7\,A_{0,2}}{\sqrt{R^4-1}}-\frac{1}{4}R\,A_{2,0}+\frac{1}{2}\frac{A_{0,2}}{\sqrt{R^4-1}\,R}$$
$$+\frac{3}{2}b_{1,1}\,R^5\,b_{0,2}-\frac{5}{4}b_{1,1}\,R\,b_{0,2}-\frac{3}{2}b_{1,1}\,R^5\,b_{2,0}-\frac{3}{2}b_{1,1}\,R^5\,a_{1,1}+\frac{3}{4}b_{1,1}\,R\,b_{2,0}+\frac{5}{4}b_{1,1}\,R\,a_{1,1}$$
$$-\frac{1}{2}R^5\,B_{1,1}-\frac{1}{2}\frac{R^7\,A_{2,0}}{\sqrt{R^4-1}}$$

```
> F2:=collect(%,R);
```

$$F2 := \left(\frac{3}{2}\frac{b_{1,1}\,a_{1,1}}{\sqrt{R^4-1}}+\frac{1}{2}\frac{A_{0,2}}{\sqrt{R^4-1}}-\frac{3}{2}\frac{b_{1,1}\,b_{0,2}}{\sqrt{R^4-1}}-\frac{1}{2}\frac{A_{2,0}}{\sqrt{R^4-1}}+\frac{3}{2}\frac{b_{1,1}\,b_{2,0}}{\sqrt{R^4-1}}+\frac{1}{2}\frac{B_{1,1}}{\sqrt{R^4-1}}\right)R^7$$
$$+\left(-\frac{3}{2}b_{1,1}\,a_{1,1}-\frac{1}{2}B_{1,1}-\frac{1}{2}A_{0,2}+\frac{1}{2}A_{2,0}-\frac{3}{2}b_{1,1}\,b_{2,0}+\frac{3}{2}b_{1,1}\,b_{0,2}\right)R^5$$
$$+\left(-\frac{2\,b_{1,1}\,a_{1,1}}{\sqrt{R^4-1}}-\frac{1}{2}\frac{B_{1,1}}{\sqrt{R^4-1}}-\frac{3}{2}\frac{b_{1,1}\,b_{2,0}}{\sqrt{R^4-1}}-\frac{A_{0,2}}{\sqrt{R^4-1}}+\frac{2\,b_{1,1}\,b_{0,2}}{\sqrt{R^4-1}}+\frac{1}{2}\frac{A_{2,0}}{\sqrt{R^4-1}}\right)R^3$$
$$+\left(-\frac{1}{4}A_{2,0}-\frac{5}{4}b_{1,1}\,b_{0,2}+\frac{1}{4}B_{1,1}+\frac{3}{4}A_{0,2}+\frac{5}{4}b_{1,1}\,a_{1,1}+\frac{3}{4}b_{1,1}\,b_{2,0}\right)R$$
$$+\frac{\frac{1}{2}\frac{A_{0,2}}{\sqrt{R^4-1}}+\frac{1}{2}\frac{b_{1,1}\,a_{1,1}}{\sqrt{R^4-1}}-\frac{1}{2}\frac{b_{1,1}\,b_{0,2}}{\sqrt{R^4-1}}}{R}$$

```
> l[4]:=collect((3/2*b[1,1]*b[0,2]+1/2*A[2,0]-1/2*B[1,1]-3/2*b[1,1]*b[2,0]-3/2*b[1,1]*a[1,1]-1/2*A[0,2])*(-4),b[1,1]);
```

$$l_4 := (6\,a_{1,1}+6\,b_{2,0}-6\,b_{0,2})\,b_{1,1}+2\,B_{1,1}+2\,A_{0,2}-2\,A_{2,0}$$

```
> l[5]:=collect((3/4*A[0,2]+3/4*b[1,1]*b[2,0]+5/4*b[1,1]*a[1,1]-5/4*b[1,1]*b[0,2]-1/4*A[2,0]+1/4*B[1,1])*(-4),b[1,1]);
```

$$l_5 := (5\,b_{0,2}-5\,a_{1,1}-3\,b_{2,0})\,b_{1,1}+A_{2,0}-B_{1,1}-3\,A_{0,2}$$

```
> l[6,1]:=collect(expand((-1/2*1/(R^4-1)^(1/2)*A[2,0]+1/2*1/(R^4-1)^(1/2)*A[0,2]+3/2*b[1,1]/(R^4-1)^(1/2)*a[1,1]+1/2*1/(R^4-1)^(1/2)*B[1,1]+3/2*b[1,1]/(R^4-1)^(1/2)*b[2,0]-3/2*b[1,1]/(R^4-1)^(1/2)*b[0,2])*(-4)*((R^4-1)^(1/2))),b[1,1]);
```

$$l_{6,1} := (-6\,a_{1,1}+6\,b_{0,2}-6\,b_{2,0})\,b_{1,1}-2\,A_{0,2}+2\,A_{2,0}-2\,B_{1,1}$$

```
> l[6,2]:=collect(expand((2*b[1,1]/(R^4-1)^(1/2)*b[0,2]-3/2*b[1,1]/(R^4-1)^(1/2)*b[2,0]-1/2*1/(R^4-1)^(1/2)*B[1,1]-1/(R^4-1)^(1/2)*A[0,2]+1/2*1/(R^4-1)^(1/2)*A[2,0]-2*b[1,1]/(R^4-1)^(1/2)*a[1,1])*(-4)*((R^4-1)^(1/2))),b[1,1]);
```

$$l_{6,2} := (8\,a_{1,1}+6\,b_{2,0}-8\,b_{0,2})\,b_{1,1}+2\,B_{1,1}+4\,A_{0,2}-2\,A_{2,0}$$

```
> l[6,3]:=collect(expand((1/2*b[1,1]/(R^4-1)^(1/2)*a[1,1]+1/2*1/(R^4-1)^(1/2)*A[0,2]-1/2*b[1,1]/(R^4-1)^(1/2)*b[0,2])*(-4)*((R^4-1)^(1/2))),b[1,1]);
```

$$l_{6,3} := (-2\,a_{1,1}+2\,b_{0,2})\,b_{1,1}-2\,A_{0,2}$$

```
> (2*l[4]*w*(1+w^2)^3+2*l[5]*w*(1+w^2)*(1-w^2)^2+l[6,1]*(1+w^2)^4+l[6,2]*(1+w^2)^2*(1-w^2)^2+l[6,3]*(1-w^2)^4);
```

$$2\left((6\,a_{1,1}+6\,b_{2,0}-6\,b_{0,2})\,b_{1,1}+2\,B_{1,1}+2\,A_{0,2}-2\,A_{2,0}\right)w\left(1+w^2\right)^3$$
$$+2\left((5\,b_{0,2}-5\,a_{1,1}-3\,b_{2,0})\,b_{1,1}+A_{2,0}-B_{1,1}-3\,A_{0,2}\right)w\left(1+w^2\right)\left(1-w^2\right)^2$$
$$+\left((-6\,a_{1,1}+6\,b_{0,2}-6\,b_{2,0})\,b_{1,1}-2\,A_{0,2}+2\,A_{2,0}-2\,B_{1,1}\right)\left(1+w^2\right)^4$$
$$+\left((8\,a_{1,1}+6\,b_{2,0}-8\,b_{0,2})\,b_{1,1}+2\,B_{1,1}+4\,A_{0,2}-2\,A_{2,0}\right)\left(1+w^2\right)^2\left(1-w^2\right)^2$$

$$+ ((-2\,a_{1,1} + 2\,b_{0,2})\,b_{1,1} - 2\,A_{0,2})\,(1 - w^2)^4$$

```
> simplify(expand(%));
```

$$-64\,b_{1,1}\,a_{1,1}\,w^4 - 2\,b_{1,1}\,b_{0,2}\,w^7 - 46\,b_{1,1}\,b_{0,2}\,w^3 + 42\,b_{1,1}\,b_{2,0}\,w^3 + 16\,b_{1,1}\,b_{0,2}\,w^2$$
$$+ 2\,w\,b_{1,1}\,a_{1,1} + 46\,b_{1,1}\,a_{1,1}\,w^5 + 2\,b_{1,1}\,a_{1,1}\,w^7 - 16\,b_{1,1}\,a_{1,1}\,w^2 + 6\,w\,b_{1,1}\,b_{2,0}$$
$$- 48\,b_{1,1}\,b_{2,0}\,w^4 + 46\,b_{1,1}\,a_{1,1}\,w^3 + 16\,b_{1,1}\,b_{0,2}\,w^6 - 24\,b_{1,1}\,b_{2,0}\,w^6 + 64\,b_{1,1}\,b_{0,2}\,w^4$$
$$+ 42\,b_{1,1}\,b_{2,0}\,w^5 - 8\,B_{1,1}\,w^2 - 16\,B_{1,1}\,w^4 - 24\,b_{1,1}\,b_{2,0}\,w^2 - 16\,b_{1,1}\,a_{1,1}\,w^6 - 2\,w\,b_{1,1}\,b_{0,2}$$
$$+ 6\,b_{1,1}\,b_{2,0}\,w^7 - 8\,B_{1,1}\,w^6 - 32\,A_{0,2}\,w^4 + 8\,A_{2,0}\,w^2 + 16\,A_{2,0}\,w^4 + 8\,A_{2,0}\,w^6 + 2\,w\,B_{1,1}$$
$$+ 14\,B_{1,1}\,w^3 + 14\,B_{1,1}\,w^5 + 2\,B_{1,1}\,w^7 - 2\,w\,A_{0,2} + 18\,A_{0,2}\,w^3 + 18\,A_{0,2}\,w^5 - 2\,A_{0,2}\,w^7$$
$$- 2\,w\,A_{2,0} - 14\,A_{2,0}\,w^3 - 14\,A_{2,0}\,w^5 - 2\,A_{2,0}\,w^7 - 46\,b_{1,1}\,b_{0,2}\,w^5$$

```
> collect(%,w);
```

$$(2\,B_{1,1} - 2\,A_{0,2} - 2\,b_{1,1}\,b_{0,2} + 2\,b_{1,1}\,a_{1,1} - 2\,A_{2,0} + 6\,b_{1,1}\,b_{2,0})\,w^7$$
$$+ (16\,b_{1,1}\,b_{0,2} - 16\,b_{1,1}\,a_{1,1} - 8\,B_{1,1} + 8\,A_{2,0} - 24\,b_{1,1}\,b_{2,0})\,w^6$$
$$+ (42\,b_{1,1}\,b_{2,0} - 46\,b_{1,1}\,b_{0,2} + 46\,b_{1,1}\,a_{1,1} + 18\,A_{0,2} - 14\,A_{2,0} + 14\,B_{1,1})\,w^5$$
$$+ (-16\,B_{1,1} - 64\,b_{1,1}\,a_{1,1} + 16\,A_{2,0} + 64\,b_{1,1}\,b_{0,2} - 48\,b_{1,1}\,b_{2,0} - 32\,A_{0,2})\,w^4$$
$$+ (42\,b_{1,1}\,b_{2,0} - 46\,b_{1,1}\,b_{0,2} + 46\,b_{1,1}\,a_{1,1} + 18\,A_{0,2} - 14\,A_{2,0} + 14\,B_{1,1})\,w^3$$
$$+ (16\,b_{1,1}\,b_{0,2} - 16\,b_{1,1}\,a_{1,1} - 8\,B_{1,1} + 8\,A_{2,0} - 24\,b_{1,1}\,b_{2,0})\,w^2$$
$$+ (2\,B_{1,1} - 2\,A_{0,2} - 2\,b_{1,1}\,b_{0,2} + 2\,b_{1,1}\,a_{1,1} - 2\,A_{2,0} + 6\,b_{1,1}\,b_{2,0})\,w$$

```
> factor(%);
```

$$-2\,w\,(w-1)^4\,(A_{2,0}\,w^2 - 3\,b_{1,1}\,b_{2,0}\,w^2 + b_{1,1}\,b_{0,2}\,w^2 + A_{0,2}\,w^2 - B_{1,1}\,w^2 - b_{1,1}\,a_{1,1}\,w^2$$
$$+ 4\,w\,b_{1,1}\,a_{1,1} - 4\,w\,b_{1,1}\,b_{0,2} + 4\,w\,A_{0,2} + A_{2,0} - 3\,b_{1,1}\,b_{2,0} + b_{1,1}\,b_{0,2} + A_{0,2} - B_{1,1}$$
$$- b_{1,1}\,a_{1,1})$$

```
> collect(-B[1,1]*w^2-b[1,1]*a[1,1]*w^2-3*b[1,1]*b[2,0]*w^2+b[1,1]
  *b[0,2]*w^2+A[0,2]*w^2+A[2,0]*w^2-4*w*b[1,1]*b[0,2]+4*w*b[1,1]*a
  [1,1]+4*w*A[0,2]-B[1,1]-b[1,1]*a[1,1]-3*b[1,1]*b[2,0]+b[1,1]*b[0
  ,2]+A[0,2]+A[2,0],w);
```

$$(-B_{1,1} - b_{1,1}\,a_{1,1} - 3\,b_{1,1}\,b_{2,0} + b_{1,1}\,b_{0,2} + A_{0,2} + A_{2,0})\,w^2$$
$$+ (4\,b_{1,1}\,a_{1,1} - 4\,b_{1,1}\,b_{0,2} + 4\,A_{0,2})\,w - B_{1,1} - b_{1,1}\,a_{1,1} - 3\,b_{1,1}\,b_{2,0} + b_{1,1}\,b_{0,2} + A_{0,2} + A_{2,0}$$

Exemple 2 :

```
> restart;
> F21:=(2*b[1,1]/(R^4-1)^(1/2)*a[1,1]-2*b[1,1]/(R^4-1)^(1/2)*b[0,2
  ]+2*b[1,1]/(R^4-1)^(1/2)*b[2,0])*R^7+(-2*b[1,1]*a[1,1]+2*b[1,1]*
  b[0,2]-2*b[1,1]*b[2,0])*R^5+(-5/2*b[1,1]/(R^4-1)^(1/2)*a[1,1]-3/
  2*b[1,1]/(R^4-1)^(1/2)*b[2,0]+5/2*b[1,1]/(R^4-1)^(1/2)*b[0,2])*R
  ^3+(-3/2*b[1,1]*b[0,2]+3/2*b[1,1]*a[1,1]+1/2*b[1,1]*b[2,0])*R+(-
  1/2*b[1,1]/(R^4-1)^(1/2)*b[0,2]+1/2*b[1,1]/(R^4-1)^(1/2)*a[1,1])
  /R;
```

$$F21 := \left(\frac{2\,b_{1,1}\,a_{1,1}}{\sqrt{R^4-1}} - \frac{2\,b_{1,1}\,b_{0,2}}{\sqrt{R^4-1}} + \frac{2\,b_{1,1}\,b_{2,0}}{\sqrt{R^4-1}}\right) R^7$$
$$+ \left(-2\,b_{1,1}\,a_{1,1} + 2\,b_{1,1}\,b_{0,2} - 2\,b_{1,1}\,b_{2,0}\right) R^5 + \left(-\frac{5}{2}\frac{b_{1,1}\,a_{1,1}}{\sqrt{R^4-1}} - \frac{3}{2}\frac{b_{1,1}\,b_{2,0}}{\sqrt{R^4-1}} + \frac{5}{2}\frac{b_{1,1}\,b_{0,2}}{\sqrt{R^4-1}}\right) R^3$$
$$+ \left(-\frac{3}{2}\,b_{1,1}\,b_{0,2} + \frac{3}{2}\,b_{1,1}\,a_{1,1} + \frac{1}{2}\,b_{1,1}\,b_{2,0}\right) R + \frac{-\frac{1}{2}\frac{b_{1,1}\,b_{0,2}}{\sqrt{R^4-1}} + \frac{1}{2}\frac{b_{1,1}\,a_{1,1}}{\sqrt{R^4-1}}}{R}$$

```
> simplify(subs(b[1,1]=1,a[1,1]=(b[0,2]+(25/12)),b[2,0]=(1/2),a[2,
  0]=1,b[0,2]=b[0,2],%));
```

$$-\frac{-124\,R^8 + 124\,R^6\sqrt{R^4-1} + 143\,R^4 - 81\sqrt{R^4-1}\,R^2 - 25}{24\sqrt{R^4-1}\,R}$$

```
> F22 :=
  (1/2*b[1,1]/(R^4-1)^(1/2)*b[0,2]-1/2*1/(R^4-1)^(1/2)*A[2,0]+1/2*
  1/(R^4-1)^(1/2)*A[0,2]-1/2*b[1,1]/(R^4-1)^(1/2)*b[2,0]+1/2*1/(R^
  4-1)^(1/2)*B[1,1]-1/2*b[1,1]/(R^4-1)^(1/2)*a[1,1])*R^7+(1/2*A[2,
  0]-1/2*B[1,1]+1/2*b[1,1]*b[2,0]-1/2*b[1,1]*b[0,2]+1/2*b[1,1]*a[1
  ,1]-1/2*A[0,2])*R^5+(1/2*1/(R^4-1)^(1/2)*A[2,0]-1/(R^4-1)^(1/2)*
  A[0,2]+1/2*b[1,1]/(R^4-1)^(1/2)*a[1,1]-1/2*1/(R^4-1)^(1/2)*B[1,1
  ]-1/2*b[1,1]/(R^4-1)^(1/2)*b[0,2])*R^3+(3/4*A[0,2]+1/4*B[1,1]-1/
  4*A[2,0]+1/4*b[1,1]*b[2,0]+1/4*b[1,1]*b[0,2]-1/4*b[1,1]*a[1,1])*
  R+1/2*1/(R^4-1)^(1/2)/R*A[0,2];
```

$$F22 := \left(\frac{1}{2}\frac{b_{1,1}\,b_{0,2}}{\sqrt{R^4-1}} - \frac{1}{2}\frac{A_{2,0}}{\sqrt{R^4-1}} + \frac{1}{2}\frac{A_{0,2}}{\sqrt{R^4-1}} - \frac{1}{2}\frac{b_{1,1}\,b_{2,0}}{\sqrt{R^4-1}} + \frac{1}{2}\frac{B_{1,1}}{\sqrt{R^4-1}} - \frac{1}{2}\frac{b_{1,1}\,a_{1,1}}{\sqrt{R^4-1}}\right) R^7$$
$$+ \left(\frac{1}{2}\,A_{2,0} - \frac{1}{2}\,B_{1,1} + \frac{1}{2}\,b_{1,1}\,b_{2,0} - \frac{1}{2}\,b_{1,1}\,b_{0,2} + \frac{1}{2}\,b_{1,1}\,a_{1,1} - \frac{1}{2}\,A_{0,2}\right) R^5$$
$$+ \left(\frac{1}{2}\frac{A_{2,0}}{\sqrt{R^4-1}} - \frac{A_{0,2}}{\sqrt{R^4-1}} + \frac{1}{2}\frac{b_{1,1}\,a_{1,1}}{\sqrt{R^4-1}} - \frac{1}{2}\frac{B_{1,1}}{\sqrt{R^4-1}} - \frac{1}{2}\frac{b_{1,1}\,b_{0,2}}{\sqrt{R^4-1}}\right) R^3$$
$$+ \left(\frac{3}{4}\,A_{0,2} + \frac{1}{4}\,B_{1,1} - \frac{1}{4}\,A_{2,0} + \frac{1}{4}\,b_{1,1}\,b_{2,0} + \frac{1}{4}\,b_{1,1}\,b_{0,2} - \frac{1}{4}\,b_{1,1}\,a_{1,1}\right) R + \frac{1}{2}\frac{A_{0,2}}{\sqrt{R^4-1}\,R}$$

```
> simplify(subs(b[1,1]=1,a[1,1]=(b[0,2]+(25/12)),b[2,0]=(1/2),a[2,
  0]=1,A[2,0]=(B[1,1]+(13/12)),A[0,2]=0,%));
```

$$\frac{R\left(-22\,R^6 + 22\,R^4\sqrt{R^4-1} + 19\,R^2 - 8\sqrt{R^4-1}\right)}{12\sqrt{R^4-1}}$$

```
> -1/24*(-124*R^8+124*R^6*(R^4-1)^(1/2)+143*R^4-81*(R^4-1)^(1/2)*R
  ^2-25)/(R^4-1)^(1/2)/R+%;
```

$$-\frac{-124\,R^8 + 124\,R^6\sqrt{R^4-1} + 143\,R^4 - 81\sqrt{R^4-1}\,R^2 - 25}{24\sqrt{R^4-1}\,R}$$

$$+\frac{R\left(-22R^6+22R^4\sqrt{R^4-1}+19R^2-8\sqrt{R^4-1}\right)}{12\sqrt{R^4-1}}$$

```
> MM20:=simplify(%);
```

$$MM20:=-\frac{5\left(-16R^8+16R^6\sqrt{R^4-1}+21R^4-13\sqrt{R^4-1}\,R^2-5\right)}{24\sqrt{R^4-1}\,R}$$

```
> solve(%,R);
```

$$\frac{\sqrt{5}}{2},-\frac{\sqrt{5}}{2}$$

```
> (-B[1,1]-b[1,1]*a[1,1]-3*b[1,1]*b[2,0]+b[1,1]*b[0,2]+A[0,2]+A[2,
  0])*w^2+(4*b[1,1]*a[1,1]-4*b[1,1]*b[0,2]+4*A[0,2])*w-B[1,1]-b[1,
  1]*a[1,1]-3*b[1,1]*b[2,0]+b[1,1]*b[0,2]+A[0,2]+A[2,0];
```

$$\left(-B_{1,1}-b_{1,1}a_{1,1}-3b_{1,1}b_{2,0}+b_{1,1}b_{0,2}+A_{0,2}+A_{2,0}\right)w^2+\left(4b_{1,1}a_{1,1}-4b_{1,1}b_{0,2}+4A_{0,2}\right)w-B_{1,1}-b_{1,1}a_{1,1}-3b_{1,1}b_{2,0}+b_{1,1}b_{0,2}+A_{0,2}+A_{2,0}$$

```
> simplify(subs(b[1,1]=1,a[1,1]=(b[0,2]+(25/12)),b[2,0]=(1/2),a[2,
  0]=1,b[0,2]=b[0,2],A[0,2]=0,A[2,0]=B[1,1]+13/12,%));
```

$$-\frac{5}{2}w^2+\frac{25}{3}w-\frac{5}{2}$$

```
> solve(%,w);
```

$$\frac{1}{3},3$$

```
> diff(MM20,R);
```

$$-\frac{5\left(-128R^7+96R^5\sqrt{R^4-1}+\frac{32R^9}{\sqrt{R^4-1}}+84R^3-\frac{26R^5}{\sqrt{R^4-1}}-26\sqrt{R^4-1}\,R\right)}{24\sqrt{R^4-1}\,R}+\frac{5\left(-16R^8+16R^6\sqrt{R^4-1}+21R^4-13\sqrt{R^4-1}\,R^2-5\right)R^2}{12\left(R^4-1\right)^{(3/2)}}+\frac{5\left(-16R^8+16R^6\sqrt{R^4-1}+21R^4-13\sqrt{R^4-1}\,R^2-5\right)}{24\sqrt{R^4-1}\,R^2}$$

```
> simplify(subs(R=(1/2)*5^(1/2),%));
```

$$\frac{5}{9}$$

```
>
>
```